BAYLOR UNIVERSITY
HANKAMER SCHOOL OF BUSINESS
DEPARTMENT OF FINANCE, INSURANCE & REAL ESTATE

Risk Management
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Problem Set 1

Name: SOLUTIONS

1. Determine the first derivative of each of the following functions:

   (a) (Constant and Power Rules): $Y = 3 + 10X + 5X^2 \Rightarrow \dfrac{dY}{dX} = 10 + 10X = 10(1 + X)$.

   (b) (Product and Power Rules):
   $Y = 2X(4 + X^3)$; here, $U(X) = 2X$ and $W(X) = 4 + X^3$
   $\therefore \dfrac{dY}{dX} = UdW/dX + WdU/dX = 2X(3X^2) + (4 + X^3)2 = 8(1 + X^3)$.

   (c) (Quotient and Power Rules):
   $Y = 3X/(4 + X^3)$; here, $U(X) = 3X$ and $W(X) = 4 + X^3$
   $\therefore \dfrac{dY}{dX} = \dfrac{WdU/dX - UdW/dX}{W^2} = \dfrac{(4 + X^3)3 - 3X(3X^2)}{(4 + X^3)^2} = \dfrac{12 - 6X^3}{(4 + X^3)^2}$.

   (d) (Quotient and Constant Rules):
   $Y = 4X/(X - 3)$; here, $U(X) = 4X$ and $W(X) = X - 3$
   $\therefore \dfrac{dY}{dX} = \dfrac{WdU/dX - UdW/dX}{W^2} = \dfrac{(X - 3)4 - 4X}{(X - 3)^2} = \dfrac{-12}{(X - 3)^2}$.

2. Determine the second derivative of the following functions:

   (a) $Y = 4 + 9X + 3X^2; \therefore dY/dX = 9 + 6X$ and $d^2Y/dX^2 = 6$.

   (b) $Y = 4X(3 + X^2) = 12X + 4X^3; \therefore dY/dX = 12 + 12X^2$ and $d^2Y/dX^2 = 24X$.

   (c) $Y = 4X(2 + X^3) = 8X + 4X^4; \therefore dY/dX = 8 + 16X^3$ and $d^2Y/dX^2 = 48X^2$.

   (d) $Y = 4/X + 3 = 4X^{-1} + 3; \therefore dY/dX = -4X^{-2}$ and $d^2Y/dX^2 = 8X^{-3}$.

3. Find the partial derivative of Y with respect to X and the partial derivative of Y with respect to Z in each of the following cases:

   (a) $Y = 10 + 3Z + 2X; \therefore \dfrac{\partial Y}{\partial Z} = 3$ and $\dfrac{\partial Y}{\partial X} = 2$.

   (b) $Y = 18Z^2 + 4X^3; \therefore \dfrac{\partial Y}{\partial Z} = 36Z$ and $\dfrac{\partial Y}{\partial X} = 12X^2$.

   (c) $Y = Z^{0.2}X^{0.8}; \therefore \dfrac{\partial Y}{\partial Z} = 0.2\left(\dfrac{X}{Z}\right)^{.8}$ and $\dfrac{\partial Y}{\partial X} = 0.8\left(\dfrac{Z}{X}\right)^{.2}$.

   (d) $Y = 3Z/(4 + X) = 3Z(4 + X)^{-1}; \therefore \dfrac{\partial Y}{\partial Z} = 3(4 + X)^{-1}$ and $\dfrac{\partial Y}{\partial X} = -3Z(4 + X)^{-2}$.

4. One very important question facing hospitals is this: How big must a hospital be (in terms of patient-days of care) to minimize the cost per patient-day? According to one well-known study, the total cost (in dollars) of operating a hospital (of a particular type) can be approximated by

$$C = 4,700,000 + 0.00013X^2,$$

where X represents the number of patient-days.

(a) Derive a formula for the relationship between cost per patient-day and the number of patient days.

SOLUTION: The cost per patient-day is the total cost divided by the number of patient-days; i.e., $Y = C/X = 4,700,000/X + 0.00013X$.

(b) On the basis of the results of this study, how big must a hospital be (in terms of patient-days) to minimize the cost per patient-day?

SOLUTION: Our objective is to select a value for X such that Y is minimized. We do this by solving the first order condition; i.e., we differentiate Y with respect to X, set the resulting equation equal to 0, and then solve for the optimal value for X:

$dY/dX = -4,700,000/X^2 + 0.00013 = 0; \therefore\ 0.00013X^2 = 4,700,000 \Rightarrow X^2 = \dfrac{4,700,000}{0.00013}; \therefore X = 190,142$ patient-days.

Thus, the cost per patient day when $X = 190{,}142$ patient-days is $Y = 4{,}700{,}000/190{,}142 + 0.00031(190{,}142) = \$49.44$. Furthermore, at this optimal (cost minimizing) scale of operation, total cost is $C = 4,700,000 + 0.00013(190,142^2) = \$9{,}400{,}017.42$.

(c) Show that your result minimizes, rather than maximizes, the cost per patient-day.

SOLUTION: Since the second order condition is satisfied; i.e., $d^2Y/dX^2 = 2(4,700,000)/X^3 > 0$, this guarantees that cost per patient day is at its minimum. Here's a picture of the cost per patient day function which confirms that 190,142 is indeed the number of patient days which minimizes cost per patient day: