Risk Aversion Class Problem Solutions

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Individual #1 has the following utility function: $U(W) = \sqrt{W}$. Her initial wealth is \$10 and she is offered a coin toss which pays off \$6 if the coin comes up heads and -\$6 if the coin comes up tails.

A. Compute the *exact* value of the certainty equivalent and of the risk premium for #1.

SOLUTION: Solving for the "exact" values of the certainty equivalent W_{CE} and the risk premium $\lambda(E(W))$ requires that we first find the expected utility of the gamble, set this equal to the utility of the certainty equivalent, and then compute W_{CE} directly. Once we know W_{CE}, then $\lambda(E(W)) = E(W) - W_{CE}$. Since the state space is $(\langle 4,16\rangle,\langle .5,.5\rangle)$, $E(W) = 10$ and $E(U(W))$ $= .5(2)+.5(4) = 3$. Therefore, $E(U(W)) = \sqrt{W_{CE}} = 3$; thus $W_{CE} = 9$, and $\lambda(E(W)) = 1$.

B. Apply the Arrow-Pratt absolute risk aversion formula to obtain an *approximation* of the risk premium for #1.

$$\lambda(E(W)) \cong \sigma^2 .5 R_A(E(W))$$

SOLUTION: The "approximate" value of $\lambda(E(W)) \cong \sigma^2 .5 R_A(E(W))$, where $R_A(E(W))$ corresponds to the ratio $-U''(W)/U'(W)$ evaluated at $E(W)$. For this utility function, $U'(W) = .5W^{-.5}$, and $U''(W) = -.25W^{-1.5}$, so $-U''(W)/U'(W) = .5/W$ and $R_A(E(W)) = .5/10 = .05$. Since the standard deviation of a fair coin toss is half of the total dispersion between the state contingent wealth values, this implies that $\sigma = 6$, which implies that $\sigma^2 = 36$. Therefore, $\lambda(E(W)) \cong 36(.5)(.05) = .9$.

C. Show that #1's absolute risk aversion is decreasing in wealth.

SOLUTION: Since $R_A(W) = .5/W$, $\dfrac{dR_A(W)}{dW} = R_A(W) = -.5/W^2$; i.e., #1's absolute risk aversion is decreasing in wealth. Although the calculus lends a nice touch, that #1's absolute risk aversion is decreasing in wealth is apparent by inspection.

D. Suppose that individual #2 is offered this gamble. Individual #2 is identical in all respects to individual #1, except #2's utility $U(W) = W^{.25}$. Compute the *exact* value of the certainty equivalent and of the risk premium for #2, and also apply the Arrow-Pratt absolute risk aversion formula to obtain an *approximation* of the risk premium for this individual.

SOLUTION: Individual #2's expected utility $E(U(W)) = .5(4^{.25})+.5(16^{.25}) = 1.707$. Therefore, $E(U(W)) = W_{CE}^{.25} = 1.707$; thus $W_{CE} = 1.707^4 = 8.49$, and the "exact" $\lambda(E(W)) = 1.51$. Since $R_A(W) = .75/W$ for Individual #2, it follows that $\lambda(E(W)) \cong 36(.5)(.075) = 1.35$.

E. Who is more risk averse, #1 or #2? Explain why.

SOLUTION: Individual #2 is more risk averse than Individual #1, since #2 has a higher risk aversion coefficient than #1. Consequently, other things equal, #2 also has a higher risk premium.