

## Midterm Exam #1 Formula Sheet

### 1. Expected value ( $E(W)$ ) and variance ( $\sigma_W^2$ )

- $E(W) = \sum_{s=1}^n p_s W_s$ , where  $p_s$  = the probability of state  $s$  and  $W_s$  = state  $s$  wealth, and
- $\sigma_W^2 = \sum_{s=1}^n p_s (W_s - E(W))^2$ .

### 2. Expected Utility ( $E(U(W))$ )

- $E(U(W)) = \sum_{s=1}^n p_s U(W_s)$ , where  $U(W_s)$  = state-contingent utility of wealth.

### 3. Certainty Equivalent of Wealth ( $W_{CE}$ ) and Risk Premium ( $\lambda$ ) - Two Methods

- “Exact” Method: Set  $E(U(W)) = U(W_{CE})$  and solve for  $W_{CE}$ ; then  $\lambda = E(W) - W_{CE}$ .
- Arrow-Pratt Method:  $\lambda = .5\sigma_W^2 R_A(E(W))$ , where  $R_A(W) = -U''(W)/U'(W)$ . Then  $W_{CE} = E(W) - \lambda$ .

### 4. Risk Pooling

- $\sigma_{L_p}^2 = \frac{\sigma^2}{n} + \frac{n-1}{n} \rho \sigma^2$ , where  $n$  corresponds to the number of identically distributed insured risks,  $\frac{\sigma^2}{n}$  corresponds to the average variance per insured risk, and  $\rho$  corresponds to the correlation between all pairwise combinations of  $n$  identically distributed insured risks.

### 5. Mean-Variance Model

If variance is a “complete” risk measure, then  $E(U(X_i)) > E(U(X_j))$  for all risk averse utility functions under the following set of conditions:

- $E(X_i) > E(X_j)$  and  $\sigma_{X_i} < \sigma_{X_j}$ ;
- $E(X_i) > E(X_j)$  and  $\sigma_{X_i} = \sigma_{X_j}$ ; and
- $E(X_i) = E(X_j)$  and  $\sigma_{X_i} < \sigma_{X_j}$ .

### 6. Stochastic Dominance Model

If  $X_i$  stochastically dominates  $X_j$ , then  $E(U(X_i)) > E(U(X_j))$  for all risk averse utility functions. Here are the formal definitions for first and second order stochastic dominance:

- First Order Stochastic Dominance: Investment  $i$  First Order Stochastic Dominates (FOSD) investment  $j$  if  $F(X_{j,s}) \geq F(X_{i,s})$  for all  $s$ .
- Second Order Stochastic Dominance: Investment  $i$  Second Order Stochastic Dominates (SOSD) investment  $j$  if  $\sum_{s=1}^n (F(X_{j,s}) - F(X_{i,s})) > 0$ .

## Standard Normal Distribution Function

<i>z</i>	<b>0.00</b>	<b>0.01</b>	<b>0.02</b>	<b>0.03</b>	<b>0.04</b>	<b>0.05</b>	<b>0.06</b>	<b>0.07</b>	<b>0.08</b>	<b>0.09</b>
<b>0.0</b>	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
<b>0.1</b>	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
<b>0.2</b>	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
<b>0.3</b>	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
<b>0.4</b>	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
<b>0.5</b>	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
<b>0.6</b>	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
<b>0.7</b>	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
<b>0.8</b>	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
<b>0.9</b>	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
<b>1.0</b>	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
<b>1.1</b>	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
<b>1.2</b>	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
<b>1.3</b>	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
<b>1.4</b>	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
<b>1.5</b>	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
<b>1.6</b>	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
<b>1.7</b>	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
<b>1.8</b>	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
<b>1.9</b>	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
<b>2.0</b>	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
<b>2.1</b>	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
<b>2.2</b>	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
<b>2.3</b>	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
<b>2.4</b>	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
<b>2.5</b>	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
<b>2.6</b>	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
<b>2.7</b>	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
<b>2.8</b>	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
<b>2.9</b>	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
<b>3.0</b>	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990

**Note:** For negative values of  $z$ , first determine the probability of positive  $z$  ( $N(z)$ ) and then calculate  $1 - N(z)$  in order to obtain  $N(-z)$ .