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pp. 1-6 Gregory Niehaus

1. Supply of Insurance

1.1 How Insurance Works

Our first objective is to understand why insurance companies are better able to bear risk than individuals. That is, why will an insurance company allow you to shift the risk of an accident? One possible answer is that insurance companies are less risk averse than others. However, it is unlikely that the owners of insurance companies are inherently less risk averse than those that seek insurance. This point is clear if you recognize that the owners of stock insurance companies are shareholders (which may include you and me) and owners of mutual insurance companies are the policyholders.

This discussion leads one to the conclusion that it is not inherent differences between individuals that allows one individual to shift risk to another. Instead, the insurance company does something to allow it to accept risks. In particular, an insurance company is able to pool risks and therefore reduce the amount of risk. To understand how risk is reduced by pooling we must examine a mathematical concept called the Law of Large Numbers (LLN).

Intuitively, the LLN states that as the sample size increases, the average outcome from the sample approaches the expected outcome of the population. In other words, the sample mean approaches the population mean. Another way of thinking about the LLN is that as the sample size increases, the variance about the expected outcome decreases.

Law of Large Numbers

Mathematically, the LLN can be stated as follows:

Let $X_1, \ldots, X_N$ be independent random variables having a common distribution with finite mean, μ, and set $S_N = \Sigma X_i$. Then, for any $\delta > 0$,

$$\lim_{N \longrightarrow \infty} P(|S_N/N - \mu| \geq \delta) = 0$$

To illustrate the LLN, flip a fair coin N times. For the i^{th} flip, let

$$X_i = \begin{cases} 1 \text{ if heads.} \\ 0 \text{ if tails} \end{cases}$$

Thus, each X_i is independent and has a mean of 0.5 (i.e., $\mu = .5$).

$$S_N = \Sigma X_i$$

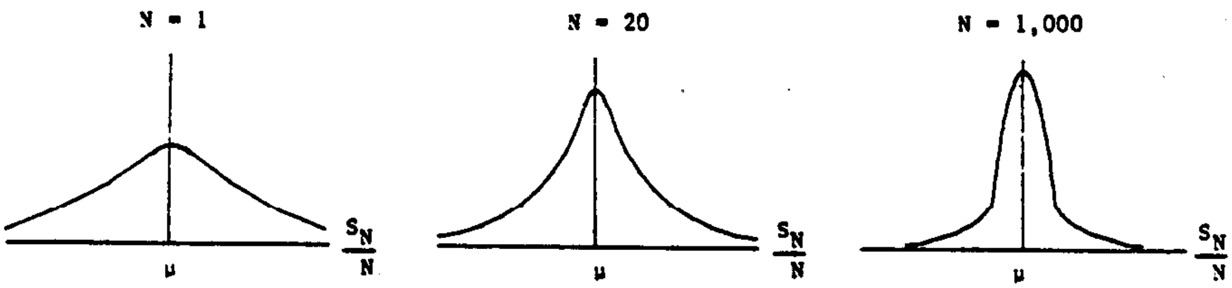
Choose δ to be small, say .001. Then the LLN implies that $P(|S_N/N - .5| \geq .001) \longrightarrow 0$ as $N \longrightarrow \infty$.

Intuition about the LLN can be obtained by considering the variance of the average outcome:

$$Var(S_N/N) = 1/N^2 \, Var(S_N) = 1/N \, Var(X_i).$$

So, as $N \longrightarrow \infty$, $Var(S_N/N) \longrightarrow 0$.

Thus, as N gets large, the variability of the average outcome gets very small. The diagrams below illustrate what happens to the distribution of S_N/N as N increases.



Thus, the LLN states that if a large number of independent identically distributed risks are pooled together then the average loss will be highly predictable.

Risk Pooling

The following example illustrates some important points about risk pooling.

Let X_1 = loss on house for person 1

X_2 = loss on house for person 2.

Assume

$$E(X_i) = \mu \quad \text{and } Var(X_i) = \sigma^2, \text{ for } i = 1, 2.$$

What if 1 and 2 get together and decide to split the outcome evenly? Then

$$Z = (X_1 + X_2)/2 = \text{outcome of each person.}$$

Then,

$$E(Z) = \mu, \text{ and}$$

$$Var(Z) = (.5)^2 \, Var(X_1) + (.5)^2 \, Var(X_2) + 2(.5)(.5) \, Cov(X_1, X_2).$$

Rewrite,

$$\text{Cov}(X_1, X_2) = \rho\sigma_1\sigma_2 = \rho\sigma^2,$$

where ρ is the correlation coefficient between X_1 and X_2.

Then, $\text{Var}(Z) = (.5)^2[\sigma^2 + \sigma^2 + 2\rho\sigma^2]$

$$= .5\sigma^2(1 + \rho)$$

If $\rho = 0$, that is, the losses are not correlated, then

$$\text{Var}(Z) = .5\sigma^2$$

In this case, risk is cut in half by getting together. If $\rho = 1$, then

$$\text{Var}(Z) = \sigma^2.$$

In this case, there is no gain to getting together. If $\rho < 1$, then

$$\text{Var}(Z) < \sigma^2.$$

In this case, there is a gain to getting together. If $\rho = -1$, then

$$\text{Var}(Z) = 0$$

In this case, getting together eliminates risk altogether. Thus, whenever losses are less than perfectly correlated, risk (variance) can be reduced by pooling.

Implications of the LLN for Insurance Pricing

Assume there are no costs associated with selling insurance and that profits are zero.

Example 1: Independent Identically Distributed Losses.

Probability of injury = .01.
Insurer pays $1,000 if illness occurs.
Let N = number of individuals insured.
Let X_i = claim of individual i.

Assume the X_i's are independent and that

$$X_i = \begin{cases} 0 \text{ w/probability} = .99 \\ 1{,}000 \text{ w/probability} = .01 \end{cases}$$

Then,

$$E(X_i) = 10$$

$$S_N = \Sigma X_i = \text{total payout by insurer}$$

$$\text{LLN} \Longrightarrow S_N/N \longrightarrow 10 \text{ in the probability limit.}$$

Assuming no transaction costs, the insurer can charge a premium equal to $10 to each individual and be relatively certain to have enough money to pay all the claims.

If N was small and the insurer got several bad draws, it could not pay all of the claims.

Thus, the LLN allows an insurer to charge premiums that are close to the expected outcome and still make the insurance credible. **IMPORTANT POINT**

A premium that is equal to the expected outcome is called an <u>actuarially fair premium</u>.

Example 2: Correlated Identically Distributed Losses.

Notice that the previous example assumes that the claims are independent from one another. Suppose instead that the risks were positively correlated. That is, claims tend to occur together. An example would be claims due to an epidemic. In this case, the insurer is not able to reduce risk to zero by pooling only these risks.

Let X_i represent the loss from policy i, i = 1, ... ,N. Assume

$$E(X_i) = \mu = \$10$$

$$Var(X_i) = \sigma^2$$

$$Cov(X_i, X_j) = \rho\sigma^2, \text{ where } \rho \text{ is the correlation coefficient.}$$

Let $S_N/N = 1/N \sum_{i=1}^{N} X_i$ = average loss per policy.

$$Var(S_N/N) = \frac{1}{N^2}\left[\sum_{i=1}^{N} \sigma^2 + \sum_{i=1}^{N} \sum_{j \neq i}^{N} \rho\sigma^2\right]$$

$$Var(S_N/N) = \frac{N\sigma^2}{N^2} + \frac{N(N-1)\rho\sigma^2}{N^2}$$

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$$Var(S_N/N) = 1/N \text{ (variance)} + \frac{N-1}{N} \text{ (covariance)}.$$

As $N \longrightarrow \infty$, $Var(S_N/N) \longrightarrow$ (covariance).

Thus, the insurer is bearing risk even if there are an infinite number of policies.

Intuitively, a bad state could occur in which a large number of the insured filed claims. Suppose 25% of the policyholders filed claims. If N = 10,000, then the insurer would have to pay (2,500)($1,000) = $2,500,000 in claims. This sum could not be paid by only charging the expected loss ($10) per customer.

This example illustrates a potential problem if an insurer sells policies against losses that are correlated. There are two possible responses for the insurer. First, the insurer can charge premiums that exceed the expected loss. In this way, the insurer will be compensated for bearing the risk that is not diversified away. Second, the insurer can attempt to diversify the risk associated with these correlated policies with other types of policies, such as fire insurance or auto insurance. Reinsurance among insurance companies facilitates the second response.

Also note that the second response is the more efficient outcome, because risk is diversified away. Consequently, competition among insurers would lead to the second outcome.

Also note that there are some risks that simply cannot be diversified away no matter how large and diverse a portfolio of policies is put together. Such risks are called systematic risks. If a risk is systematic, there are likely to be losses to all policies at the same time. A nuclear war would be one example.

A problem arises with insuring systematic risk. A loss that is systematic occurs to all people at the same time. Thus, these types of risks cannot be credibly insured by paying premiums to an insurance company each year because the loss may occur in an early year. Since the loss occurs in an early year, the insurer has not collected enough yearly premiums to pay out all the claims. To make the insurance credible, the insurer would have to put up capital and invest the funds in risk free securities in order for consumers to believe they will be covered if a loss occurs. The insurer would have to be compensated (in terms of higher premiums) for the risk that all of the capital is lost. In other words, the insurer would have to be compensated for the systematic risk that it is bearing. Notice the similarity to the Capital Asset Pricing Model from the finance literature.

Systematic risks may be well suited to be insured by a stable government. If a loss occurs in an early year the government can pay the claims by borrowing money and levying taxes in future years.