

BAYLOR UNIVERSITY  
HANKAMER SCHOOL OF BUSINESS  
DEPARTMENT OF FINANCE, INSURANCE & REAL ESTATE

Risk Management  
Dr. Garven  
Problem Set 8

Name: \_\_\_\_\_ SOLUTIONS \_\_\_\_\_

**Problem 1** (40 points)

Suppose the current value of a (non-dividend-paying) stock is \$1,000, and the annual continuously compounded riskless rate of interest is 3%. Based on the example provided on pp. 9-13 from the “[Derivatives Theory, Part 1](#)” lecture note, solve parts A and B below.

- A. (20 points) What is the “arbitrage-free” price for a forward contract on this stock which matures 1 year from today?

SOLUTION:  $F = Se^{rT} = \$1,000e^{.03} = \$1,030.45$

- B. (20 points) Suppose the forward price is \$1,040. Describe a profitable zero risk, zero net investment trading strategy involving the forward contract and its replicating portfolio. If you implement such a strategy, how much profit will you earn?

SOLUTION: Since the forward price of \$1,040 exceeds its “arbitrage-free” price by \$9.55, it is too expensive. In order to take advantage of this mispricing in a way that involves zero risk and zero net investment, one should sell a 1-year forward contract for \$1,040, buy the stock for \$1,000, and fund the stock purchase with \$1,000 in (riskless) debt. The following table succinctly summarizes this trading strategy:

Transaction	Payoff now	Payoff @ $T$
Sell Forward	\$0	$\$1,040 - S_T$
Buy Stock	(\$1,000)	$S_T$
Borrow	\$1,000	$(\$1,000)e^{.03} = (\$1,030.45)$
<b>Arbitrage Profit</b>	<b>\$0</b>	<b>\$9.55</b>

**Problem 2** (60 points)

The price of a share of Zoom stock is currently \$50. It is known that at the end of 1 year, the Zoom share price will be either \$62.50 or \$40. The riskless interest rate is 2% per year.

- A. (10 points) Calculate the price of a 1-year European call option on Zoom stock with an exercise price of \$50 by applying the replicating portfolio approach.

SOLUTION: According to the replicating portfolio approach:

$$\begin{aligned}
 C_u &= \text{Max}(0, S_u - K) = 12.50 \\
 C_d &= \text{Max}(0, S_d - K) = 0 \\
 V_{RP} &= \Delta S + B \\
 \Delta S &= \frac{C_u - C_d}{S(u - d)} S = \frac{12.50}{50(.45)} 50 = .5556(50) = 27.78 \\
 B &= \frac{uC_d - dC_u}{e^{r\delta t}(u - d)} = \frac{.25(0) - .2(12.50)}{e^{.02(1)}(.45)} = -21.78 \\
 \therefore C = V_{RP} &= \Delta S + B = \$27.78 - \$21.78 = \$6.00.
 \end{aligned}$$

- B. (10 points) Calculate the price of a 1-year European call option on Zoom stock with an exercise price of \$50 by applying the delta hedging approach.

SOLUTION: According to the Delta Hedging Approach:

$$\begin{aligned}
 V_H &= C - \Delta S = C - \Delta 50. \\
 V_H^u &= V_H^d \Rightarrow 12.50 - \Delta 62.50 = 0 - \Delta 40 \Rightarrow \Delta = .5556. \\
 V_H^u &= V_H^d = -22.22 \\
 V_H &= C - \Delta 50 = C - 27.78 = -e^{-.02} 22.22 = -.9802(22.22) = -21.78 \\
 \therefore C &= \$6.00
 \end{aligned}$$

- C. (10 points) Calculate the price of a 1-year European call option on Zoom stock with an exercise price of \$50 by applying the risk neutral valuation approach.

SOLUTION: The risk neutral probability of an up move is  $q = \frac{e^{r\delta t} - d}{u - d} = \frac{e^{.02} - .8}{1.25 - .8} = .4893$ . Since the stock is worth  $50(1.25) = \$62.50$  at the  $u$  node and  $50(.8) = \$40$  at the  $d$  node, this means that the call is only in the money at the  $u$  node; specifically, it is worth \$12.50 at that node. Therefore, the price of a one-year call option is

$$C = e^{-r\delta t}[qC_u + (1 - q)C_d] = e^{-.02} [.4893(12.50)] = \$6.00.$$

- D. (10 points) Calculate the price of a 1-year European put option on Zoom stock with an exercise price of \$50.

SOLUTION: According to the put-call parity equation,  $C + Ke^{-r\delta t} = P + S$ ; therefore,  $P = C + Ke^{-r\delta t} - S \Rightarrow P = 6 + 50e^{-.02(1)} - 50 = \$5.01$ .

- E. (20 points) Next, add another 1-year timestep; i.e., it is known that at the end of 2 years, the Zoom share price will be \$78.13, \$50, or \$32. Calculate the price of a 2-year European call option on Zoom stock with an exercise price of \$50. Also calculate the price of a 2-year European put option on Zoom stock with an exercise price of \$50.

SOLUTION: Note that the call option is only in the money at the  $uu$  node, where it is worth \$28.13. It is worthless at the  $ud$  and  $dd$  nodes. Applying risk neutral valuation,

the price of a two-year call option is

$$\begin{aligned} C &= e^{-2r\delta t}[q^2C_{uu} + 2q(1-q)C_{ud} + (1-q)^2C_{dd}] \\ &= e^{-.04} [.4893^2(28.13)] = \$6.47. \end{aligned}$$

Regarding the (otherwise identical) 2-year put option, put-call parity indicates that the put is worth \$4.51:

$$P = C + Ke^{-2r\delta t} - S = 6.47 + \$50e^{-.04} - \$50 = \$4.51.$$