

Option Pricing Class Problem Solutions

Finance 4335, November 9, 2021

Bitcoin, Inc., stock is currently worth \$56. Each year, it can change by a factor of 0.9 or 1.3. The stock pays no dividends, and the annual continuously compounded risk-free interest rate is 4%.

- A. Calculate the price of a one-year European put option on Bitcoin, Inc. stock with an exercise price of \$60.

SOLUTION: We can solve this problem by via replicating portfolio, delta hedging and risk neutral valuation approaches.

1. According to the Replicating Portfolio Approach:

$$\Delta = \frac{P_u - P_d}{uS - dS} = \frac{0 - 9.60}{72.80 - 50.40} = -.4286; \text{ and } B = \frac{uP_d - dP_u}{e^{r\delta t}(u - d)} = \frac{1.3(9.60) - .9(0)}{1.0408(.4)} = 29.98. \text{ Then } V_{RP} = P = \Delta S + B = -.4286(56) + 29.98 = \$5.98.$$

2. According to the Delta Hedging Approach:

$$\begin{aligned} P_u &= \text{Max}(0, K - S_u) = 0 \\ P_d &= \text{Max}(0, K - S_d) = 9.6 \\ V_H &= P + \Delta S = P + \Delta 56. \\ V_H^u &= V_H^d \Rightarrow 0 + \Delta 72.80 = 9.60 + \Delta 50.40 \Rightarrow \Delta = .4286. \\ V_H^u &= V_H^d = 31.20 \\ V_H &= P + \Delta 56 = P + 24 = e^{-.04} 31.20 = .9608(31.20) = 29.98 \\ P &= \$5.98 \end{aligned}$$

3. According to the Risk Neutral Valuation Approach:

The risk neutral probability of an up move is $q = \frac{e^{r\delta t} - d}{u - d} = \frac{e^{.04} - .9}{1.3 - .9} = .352$. Since the stock is worth $\$56(1.3) = \72.80 at the u node and $\$56(.9) = \50.40 at the d node, this means that the put is only in the money at the d node; specifically, it is worth \$9.60 at that node. Therefore, the price of a one-year put option is

$$p = e^{-r\delta t}[qp_u + (1 - q)p_d] = e^{-.04} [.648(9.60)] = \$5.98.$$

- B. Calculate the price of a one-year European call option on Bitcoin, Inc. stock with an exercise price of \$60.

SOLUTION: We can solve this problem by via put-call parity, delta hedging, replicating portfolio, and risk neutral valuation approaches.

1. According to put-call parity,

$$C = P + S - Ke^{-r\delta t} = \$5.98 + \$56 - \$60e^{-.04} = \$4.33.$$

2. According to the Replicating Portfolio Approach:

$$\Delta = \frac{C_u - C_d}{uS - dS} = \frac{12.80 - 0}{72.80 - 50.40} = .5714 \quad \text{and} \quad B = \frac{uC_d - dC_u}{e^{r\delta t}(u - d)} = \frac{1.3(0) - .9(12.80)}{1.0408(.4)} = -27.67.$$

Then $V_{RP} = C = \Delta S + B = .5714(56) - 27.67 = \$4.33.$

3. According to the Delta Hedging Approach:

$$\begin{aligned} C_u &= \text{Max}(0, S_u - K) = 12.80 \\ C_d &= \text{Max}(0, S_d - K) = 0 \\ V_H &= C - \Delta S = C - \Delta 56. \\ V_H^u &= V_H^d \Rightarrow 12.80 - \Delta 72.80 = 0 - \Delta 50.40 \Rightarrow \Delta = .5714. \\ V_H^u &= V_H^d = -28.80 \\ V_H &= C - \Delta 56 = C - 32 = -e^{-.04} 28.80 = -.9608(28.80) = -27.67 \\ C &= \$4.33 \end{aligned}$$

4. According to the Risk Neutral Valuation Approach:

The risk neutral probability of an up move is $q = \frac{e^{r\delta t} - d}{u - d} = \frac{e^{.04} - .9}{1.3 - .9} = .352.$ Since the stock is worth $\$56(1.3) = \72.80 at the u node and $\$56(.9) = \50.40 at the d node, this means that the call is only in the money at the u node; specifically, it is worth \$12.80 at that node. Therefore, the price of a one-year call option is

$$c = e^{-r\delta t}[qc_u + (1 - q)c_d] = e^{-.04} [.352(12.80)] = \$4.33.$$

C. Calculate the price of a two-year European put option on Bitcoin, Inc. stock with an exercise price of \$60.

SOLUTION: With two timesteps, there will be 3 terminal nodes – uu , ud , and dd . The share price at these 3 nodes is \$94.64, \$65.52, and \$45.36 respectively. This implies that the put is only in the money at the dd node; specifically, it is worth \$14.64 at that node. Therefore, the price of a two-year put option is

$$p = e^{-2r\delta t}[q^2 p_{uu} + 2q(1 - q)p_{ud} + (1 - q)^2 p_{dd}] = e^{-.08} (.648^2)(14.64) = \$5.67.$$

D. Calculate the price of a two-year European call option on Bitcoin, Inc. stock with an exercise price of \$60.

SOLUTION: We can solve this problem by either applying the put-call parity equation or risk neutral valuation. According to put-call parity,

$$c = p + S - Ke^{-2r\delta t} = \$5.67 + \$56 - \$60e^{-.08} = \$6.29.$$

Applying risk neutral valuation, note that the call option is in the money at the uu node (where it is worth $\$94.64 - 60 = \34.64) and at the ud node (where it is worth $\$65.52 - 60 = \5.62); however, it is out of the money at the dd node. Therefore, the price of a two-year call option is

$$c = e^{-2r\delta t}[q^2c_{uu}+2q(1-q)c_{ud}+(1-q)^2c_{dd}] = e^{-.08} [.352^2(34.64)+2(.352)(.648)(5.62)] = \$6.29.$$