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CHAPTER 5
CAPITAL MARKET THEORY

The corporate objective assumed for this book is that a firm wishes to maximize the value of its owners' equity. This is equivalent to maximizing the value of the firm's shares. To derive risk management strategies in pursuit of this objective requires some knowledge of how stock prices are determined. In particular, we must examine how the capital market functions and what motivates investors in their decisions to purchase securities. In the conclusion of the previous chapter, we examined the possibility for diversification across the various risky activities undertaken by a firm. The effect of pooling the various risky cash flows is to reduce the total level of risk in a firm's earnings. These earnings accrue to the owners of the firm and the owners must absorb any remaining risk. How does this remaining risk affect the welfare of the shareholders'? Presumably, the shareholders' main concern is with the value of their shares. So we may restate the last question. How does the remaining risk in the firm's earnings affect the share values? It is tempting to reply that shareholders are risk-averse and therefore risk will simply reduce share value. Therefore, any reduction in risk, perhaps by risk management strategy, will increase share value. This straightforward total-risk approach is widespread in the traditional risk management literature. However, we will see that it is an oversimplification that results in misleading and sometimes incorrect conclusions. The possibilities for diversification have not yet been exhausted.

A CAPITAL MARKET IN WHICH INVESTORS HOLD ONLY ONE SECURITY

It is common, and probably not unreasonable, to assume that most people are averse to risk and that this attitude is reflected in many forms of economic behavior. The most obvious *prima facie* evidence in support of this assumption lies in the widespread demand for personal insurance protection. Most of us appear willing to convert the uncertain prospect of a large loss of wealth through the destruction of our home or our life into a regular payment of an insurance premium, even though insurance is costlier in the long run, since premiums typically exceed the expected value of loss. There is considerable evidence that investors (if indeed they form a separate group from homeowners) typically are averse to risk and that this attitude influences their investment strategies and thereby helps to determine the prices of securities.

Investors prefer higher returns on their investment holdings to lower returns, other things being equal. A slightly stronger assumption is that investors prefer less to more risk, other things being equal. Together, these assumptions are broadly acceptable as a general statement of investor motivation. It is conventional to measure return and risk as the expected value of the rate of return and its standard deviation. The rate of return yielded on an investment in a share of stock is the sum of the dividend payment and the capital gain, expressed as a ratio of the price initially paid for the stock. Thus, the actual rate of return is

$$r_t = \frac{d_{t+1} + (P_{t+1} - P_t)}{P_t}$$

where r_t = the return from time t to t+1
d_{t+1} = the dividend payable at t+1
P_t = the share price at t
P_{t+1} = the share price at t+1

The expected rate of return for any period t is denoted

$$E(r_t) = \sum_i p_i \, r_{i,t}$$

where p_i is the probability of return r_i in period t. The standard deviation is

$$\sigma(r_t) = \left\{E[r_{i,t} - E(r_t)]^2\right\}^{1/2}$$

Now consider an investor who is faced with a choice between alternative investments and who seeks to base his or her decision on expected return and standard deviation. For our example, we will consider just three possible stocks from which to choose and, to keep things simple, we will further assume that the expected return and its standard deviation can be estimated from recent past experience. More specifically, the mean annual rate of return over the recent 10 time periods, and its standard error, are used as estimators for future returns. As shown in Table 1, an examination of the three stocks reveals that stock C has the highest mean return and, therefore, is estimated to have the highest expected return. Stock B has the lowest expected return, with stock A having an expected return roughly halfway between stocks B and C. In terms of risk, stock B clearly performs badly, having a much higher standard deviation than stocks A and C. Stock C is a little riskier than stock A.

TABLE 1
ACTUAL RATES OF RETURN

		Stock		
	Period	**A**	**B**	**C**
	1	0.06	0.03	0.13
	2	-0.03	0.33	- 0.07
	3	0.00	0.23	0.05
	4	0.06	-0.12	0.13
	5	0.20	-0.22	0.18
	6	0.13	-0.15	0.13
	7	0.10	0.19	0.08
	8	-0.06	0.24	-0.10
	9	0.05	0.16	0.10
	10	0.23	0.00	0.18
Mean return, E (r)	0.074	0.069	0.081	
Standard error, σ(r)		0.089	0.179	0.091

Our intrepid investor must now make his or her choice. If the investor has to choose one stock, he or she will clearly avoid stock B because it has the lowest return and highest risk. So the choice is between stocks A and C. Stock C exhibits higher return and higher risk than stock A. If the individual is highly risk averse, he or she will probably choose stock A, being more impressed by its lower risk and being willing to sacrifice a little in terms of expected return. However, if the investor is not so intensively averse to risk (although still risk averse), he or she may decide that stock C offers a distinct improvement in expected return over stock A with only a small increase in risk. Under such circumstances, stock C is preferred to stock A. Thus the ranking of the three stocks, using the symbol > to denote preference, is

A > C > B	for highly risk averse
C > A > B	for less risk averse

Figure 1 shows the risk-return characteristics of the stocks. Stock C has the highest return and stock B the lowest. Stock B has the highest risk and stock A the lowest. In the space shown in the figure, the investor wishes to be as far in a "northwesterly" direction as possible, since movement in this direction signifies increasing return and falling risk. On this criterion, stock B is clearly dominated by both stocks A and C, but stocks A and C cannot be ranked unambiguously because they fall in a "southwest-northeast" array with respect to each other.

Let us now see how the capital market would function to determine the price of securities. If investors were only allowed to hold one stock, nobody would wish to hold stock B and all investors would seek stocks A or C. Those already holding stock B would try to sell, thereby driving down its price. Simultaneously, investors would seek to purchase stocks A and C, thereby driving up the price. Since the rate of return for a security is based on its price, a fall in price will push up the expected rate of return, assuming no change in the firm's expected earnings. Conversely, an increase in price will lead to a fall in expected return, *ceteris paribus*. This supply-demand pressure will restructure the risk-return characteristics of the three stocks, perhaps until each of the stocks matches the preferences of some subgroup of investors. The final position might look like that shown in Figure 2, which reveals that expected return is positively related to standard deviation.

Some of the older theories of capital markets did indeed produce results like those displayed by Figure 2. This pricing structure certainly is based on risk aversion, but modem capital market theory recognizes that the assumption of risk aversion also leads the investor to hold a portfolio rather than a single security, and this radically affects the capital market equilibrium.

CAPITAL MARKET EQUILIBRIUM WITH DIVERSIFICATION

Simple Diversification

Table 2 reproduces the information concerning stocks A, B, and C, but it also includes a fourth asset D. It is seen immediately that D dominates stocks A and B in terms of risk and return and that it compares favorably with stock C. In fact, asset D delivers an expected return that is only slightly lower than that for stock C, but asset D has a much lower standard deviation. Therefore, most moderately risk averse or highly risk averse investors probably would prefer asset D to stock C. Where did asset D come from? Asset D is a portfolio comprising securities C and B.

Let us return to the problem of selecting from securities A, B, and C without the constraint that the investor hold only one security. Suppose two securities can be held, thereby forming a portfolio. It is tempting to return to the ranking derived earlier, which showed stocks A and C to be superior to stock B, although the ranking between stocks A and C was unclear. From this ranking, it appears that no rational investor would ever choose stock B.

TABLE 2
ACTUAL RATES OF RETURN

	Period	A	B	C	D
	1	0.06	0.03	0.13	0.097
	2	-0.03	0.33	-0.07	0.063
	3	0.00	0.23	0.05	0.11
	4	0.06	-0.12	0.13	0.047
	5	0.20	-0.22	0.18	0.047
	6	0.13	-0.15	0.13	0.037
	7	0.10	0.19	0.08	0.116
	8	-0.06	0.24	-0.10	0.013
	9	0.05	0.16	0.10	0.120
	10	0.23	0.00	0.18	0.120
Expected return		0.074	0.069	0.081	0.077
Standard error σ_n		0.089	0.179	0.091	0.038

Covariances,	$\sigma_{A,B}$ = -0.0117 ;	$\sigma_{A,C}$= 0.007 ;	$\sigma_{B,C}$ = -0.0131
Correlation coefficients	$r_{A,B}$ = -0.8 ;	$r_{A,C}$ = 0.86 ;	$r_{B,C}$ = -0.73

Note: D = 0.33 B + 0.67 C

However, this view ignores one important dimension. When risky cash flows are combined into a portfolio, the riskiness of the portfolio depends not only on the individual variances or standard deviations of the component stocks, but also on their covariances or correlations. Negative covariances can have a dramatic effect by reducing portfolio risk, even if the items concerned each exhibit a high level of risk. Re-examination of stocks A, B, and C reveals that stock B has the attractive property that it appears to vary inversely with both stocks A and C. When stocks A and C deliver high returns, stock B tends to deliver low returns, and vice versa. However, stocks A and C appear to vary in the same direction, suggesting positive correlation. Although stocks A and C appear to be attractive when each is considered in isolation, a portfolio comprising these two securities would not be effective in reducing portfolio risk. A portfolio comprising stocks B and C would contain securities whose returns tended to vary in opposition, thereby producing a fairly stable portfolio return. Asset D is such a portfolio, and it is formed by permitting the investor to invest one-third of his or her capital in stock B and two-thirds in stock C. The resulting portfolio has an expected return between that of stocks B and C, but a standard deviation that is much lower than that of either stock B or stock C.

The behavior of security portfolios closely parallels the portfolio behavior identified in the previous chapter. It is necessary to modify the formulas used there to calculate the portfolio mean and variance, for here we are not dealing with dollar values but with rates of return. The expected return, the variance of that rate of return, and its standard deviation for a portfolio of n securities as follows:

$$(1) \qquad E(r) = \sum_{i=1}^{n} w_i \, E(r_i)$$

$$(2) \qquad \sigma^2(r) = \sum_{i=1}^{n} w_i^2 \, \sigma_i^2 + \sum \sum_{i \neq j} w_i \, w_j \, \sigma_{i,j}$$

$$(3) \qquad \sigma(r) = \left[\sigma^2(r)\right]^{1/2}$$

where w_i = the weight of security i in the investor's portfolio
r_i = the rate of return on security i
σ_i = the standard deviation of the rate of return on security i
$\sigma_{i,j}$ = the covariance between the returns on securities i and j

Often, it is more useful to use the correlation coefficient instead of the covariance. This is easily accomplished by recalling that the correlation coefficient $r_{i,j}$ is defined by

$$r_{i,j} = \frac{\sigma_{i,j}}{\sigma_i \sigma_j}$$

Substitution into previous equation for portfolio variance is straightforward.

$$(2a) \qquad \sigma^2(r) = \sum_{i=1}^{n} w_i^2 \, \sigma_i^2 + \sum \sum_{i \neq j} w_i \, w_j \, r_{i,j} \, \sigma_i \, \sigma_j$$

The means, standard deviations, covariances, and correlation coefficients are shown at the bottom of Table 2. Consider portfolio D, comprised of one-third of capital invested in stock B and the two-thirds invested in stock C. The expected return and standard deviation can be confirmed by equations (1) and (3):

$$E(r_D) = 0.33(0.069) + 0.67(0.081) = 0.077$$

$$\sigma(r_D) = [(0.33)^2 (0.179)^2 + (0.67)^2 (0.091)^2 + 2(0.33)(0.67)(-0.0131)]^{1/2} = 0.038$$

The means and standard deviations of stocks B and C are now plotted in Figure 3(a). By forming the portfolio from stocks B and C, the investor can also choose the risk-return combination shown as D on the graph. However, these possibilities do not exhaust the investor's choice. The investor could have invested 50% in each of stocks B and C, or perhaps 75% in stock B and 25% in stock C, and so on. The various options are plotted along the broken line traveling through B and C but which is convex to the vertical axis. The fact that the line is convex clearly illustrates possibilities for reducing risk by forming a portfolio.

From earlier analysis, it should be clear that the facility for reducing risk depends on the covariance between the securities. Suppose a portfolio, labeled E, was formed by combining stocks A and C. Since these stocks are positively correlated, there should be no dramatic reduction in risk. Figure 3(b) shows, on a similar diagram to Figure 3(a), a broken line giving the risk-return combinations for portfolios of stocks A and C. Position E shows weights of 0.5 on each of stock A and stock C, with $E(r_E)$ and $\sigma(r_E)$ calculated as follows:

$$E(r_E) = 0.5(0.074) + 0.5(0.081) = 0.0775$$

$$\sigma(r_E) = [(O.5)^2(0.089)^2 + (0.5)^2 (0.091)^2 + 2(0.5)(0.5)(0.007)]^{1/2} = 0.087$$

Figures 3(a) and (b) illustrate the effect of correlation on the potential for risk reduction through portfolio formation. The more pronounced convexity in part (a) indicates much greater risk reduction than that shown in part (b). The limiting cases are given by perfect positive correlation (the correlation coefficient is unity) and perfect negative correlation (the correlation coefficient is minus unity). Although none of the pairs of securities in our example meets these criteria, figures 3(c) and (d) shows what the portfolio possibilities would be. Notice that with perfect positive correlation there is no curvature in the portfolio line at all. This is consistent with the analysis in the previous chapter, which revealed no prospect for diversification under these circumstances. With perfect negative correlation, the portfolio line actually touches the vertical axis, showing that one portfolio exists that will remove all risk. (It might be pointed out that with securities such as stocks, this very attractive perfect negative correlation is rarely found).

Derivation of the Efficient Frontier

So far, the portfolios have only been formed using pairs of securities. Relatively little imagination is needed to realize that further possibilities for diversification might exist by combining three or more securities into a portfolio. The equations given for calculating expected return and variance are not restricted to any number of securities, and we could search through the wide selection of stocks and other financial assets available to us until we found our preferred risk-return combination. This may prove to be a long search, but it is quite simple to represent what might happen.

In Figure 4, a set of stocks is represented, each by its expected return and standard deviation. For any pair of stocks, we could construct portfolio lines such as those shown in Figure 3, and the shape of each line would depend on the correlation coefficient. Only two such portfolio lines are shown to avoid congestion. However, portfolios could be formed with three or more stocks. These can be represented as "portfolios of portfolios". Thus, the thin solid line joining the broken lines AB and CD represents some of the portfolios that can be formed by combining securities A, B, C, and D. It is clear that portfolios along this line would not be attractive, since there are still other securities offering higher return and lower risk, such as stocks E and F. However, even further prospects for improvement arise from combining stocks E and F into a portfolio and perhaps including stock C and/or stock D. When all such possibilities are exhausted, the resulting envelope curve, i.e., the curve enclosing all the portfolio lines, will look like the thick solid line. The upper left portion of this envelope is known as the *efficient frontier* because it represents the most efficient portfolios that can be formed. The portfolios represented along this line deliver the minimum level of risk for each given level of expected return.

The various positions along the "northwest" segment of the efficient frontier represent portfolio choices that are efficient in mean-variance terms. Apparently, there is no optimal portfolio along this line (a statement that will be revised in due course). The more risk averse investors presumably would choose portfolios, such as

X, that exhibit relatively little risk but have a corresponding low level of expected return. More speculative investors would accept more risk to deliver higher expected return, such as portfolio Y. According to individual preference, each investor would locate at a position reflecting his or her degree of risk aversion.

To derive the efficient frontier in practice would appear to be an enormous task. There are hundreds of securities to choose from, and we have to consider every possible combination. Bearing in mind that the number of securities to be assembled into portfolios is not fixed and that the weights for each security can assume any value (subject to their summing to unity), the number of possible portfolios is infinite. This implies a pretty long game of trial and error; in fact, the proverbial three monkeys might well randomly type out the complete works of Shakespeare earlier than trial and error would lead to complete specification of the efficient frontier. However, there are easier methods. One possibility is to use quadratic programming, since its application is straightforward with a personal computer. Another possibility is to solve mathematically, since an analytical solution has been derived by Merton (1972). Thus the technology to solve the efficient frontier is now routinely available.

The efficient frontier solution to the problem of portfolio selection was presented in 1952 by Markowitz. This work represented an important turning point in capital market theory, because it offered a usable quantitative solution to the issue of portfolio selection. Its implications have proved to be far-reaching, leading to a "revolution" in the way we view the workings of the capital market.

The Market Portfolio

The efficient frontier turns out to be a misnomer, for there may be ways of improving investment performance as measured by risk and expected return. Let us suppose that, in addition to the various risky securities that are used to derive the efficient frontier, there is also a riskless security. This may be represented as debt that is free of default risk and has a fixed yield. As a rough approximation, a Federal Treasury Bill may be considered to be risk-free, Given the absence of risk, we would expect such a security to bear a fairly low yield, which is denoted r_f in Figure 5. The diagram also reproduces the efficient frontier.

The rate r_f is the risk free rate, and we will assume that unlimited borrowing and lending take place at this rate.[1] With borrowing and lending available at r_f, the investor may chose not to invest all his or her capital in risky securities. Instead, he or she may choose some portfolio, such as N, and invest half of the capital in N and the remainder in the risk free security yielding r_f. This composite position will yield an expected return and standard deviation shown as X on the straight line connecting r_f and N. Indeed, any other position on the line is also available by varying the proportions of capital invested in N and r_f. A more aggressive investor may choose to borrow money at r_f, thereby permitting him or her to invest more than the initial capital in portfolio N. This leveraged investor will achieve some position on the broken line such as Y.

These two investment strategies are illustrated quite easily by considering a lending investor who lends half her capital at the risk-free rate and a leveraged investor who borrows an amount equal to half his initial capital to invest in a risky portfolio. In this example, the risk-free rate is 0. 1; the expected return on the risky portfolio is 0. 15 with a standard deviation of 0.2 The covariance between r_f and the risky portfolio must be zero, since the former has no risk.

Lending Investor

$$E(r) = 0.5(0.1) + 0.5(0.15) \qquad = 0.125$$
$$\sigma(r) = [(0.5)^2(0)^2 + (0.5)^2(0.2)^2 + 2(0.5)(0.5)(0)]^{1/2} \qquad = 0.1$$

Leveraged Investor

$$E(r) = -0.5(0.1) + 1.5(0.15) \qquad = 0.175$$
$$\sigma(r) = [(-0.5)^2(0)^2 + (1.5)^2(0.2)^2 + 2(-0.5)(1.5)(0)]^{1/2} \quad = 0.3$$

In this example, the investor generated new opportunities not available with the efficient frontier. Portfolios along the segment $r_f N$ may well be chosen over positions on the efficient frontier. However, a leveraged position such as Y clearly is suboptimal, since there are positions on the frontier offering higher expected return for the same level of risk.

By choosing another risky portfolio on the efficient frontier and then lending or

[1]The reader should not get nervous about simplifying assumptions here. Indeed, several more simplifying assumptions will soon be made to proceed with the analysis. These assumptions are necessary to construct a predictive model, and this model should be appraised on its predictive power rather than on the accuracy of its assumptions. The assumptions can be relaxed, causing some modification of the model; however, the basic insights of the simple model are still preserved.

leveraging, the investor can do even better. This portfolio is defined by a point of tangency between the efficient frontier and a straight line going through r_f. The tangency point defines a risky portfolio that is denoted M. Lending positions are defined along the segment r_f M, and leveraged positions are defined along the segment MZ. The line r_f MZ is known as the *capital market line.* Notice that the capital market line dominates the lending/leveraged positions that could be attained holding portfolio N. For every position on r_fNY, there are corresponding positions on r_f MZ offering higher returns for the same risk or, equivalently, offering lower risk for the same level of expected return. It is also fairly obvious that the tangency portfolio M is optimal in the sense that it offers lending/leverage opportunities that dominate those offered by any other risky portfolio on the efficient frontier.

This analysis leads to the conclusion that there is a single portfolio of risky securities that is optimal for all investors regardless of personal attitude toward risk. This portfolio is M. Investors who are risk averse are better served by holding part of their capital in M and lending at the risk free rate to achieve a position on the lending portion of the capital market line than by holding a portfolio such as N that is fairly low in risk. Similarly, investors who are more speculative are better off borrowing at r_f and investing the proceeds, together with initial capital, in risky portfolio M. This leads to a position on the leveraged part of the capital market line that is superior to holding a risky - high return portfolio such as Y. Consequently, all investors, regardless of risk preference, would choose the same portfolio of risky assets, but they would differ in their financing decisions. On the latter decisions, some investors will achieve lending positions and others will achieve leveraged positions to match their individual risk preferences. This conclusion can be given as a *separation theorem*:

> *The selection of a portfolio of risky stocks is independent both of the financing decision and of individual risk preference given the assumptions of the analysis.*

If all investors are predicted to hold the same portfolio of risky stocks, then all stocks represented on the market must find their way into that portfolio. The portfolio can be none other than the *market portfolio,* which includes all traded stocks, with each stock being held in the proportion its market value bears in relation to the total market value of all stocks. This is quite a dramatic conclusion, and it leads to interesting conclusions about the way in which securities are priced. This pricing process will now occupy our attention.

Systematic and Unsystematic Risk

Some assumptions have been necessary to get this far in the examination of capital markets. Further assumptions are necessary in order to continue. These assumptions are now collected together, and from these, a model of asset pricing will be constructed:

1. Unlimited borrowing and lending can take place at a single risk-free interest rate. The Federal Treasury Bill rate usually is used as a proxy for this rate.

2. All assets can be bought and sold immediately in any quantity at the prevailing market price.

3. Assets are perfectly divisible; the investor can buy or sell any quantity, including fractions, of assets and short sales.

4. There are no taxes or transaction costs.

5. All investors base decisions only on the expected return and its variance (or standard deviation) and seek the minimum variance for any given level of return.

6. The planning horizon for investment decisions is one period. Note that the return during this period includes the terminal value of the asset.

7. Investors share identical expectations about the probability distributions of available assets.

These assumptions clearly do not accurately describe the real world. However, the relevant question is whether the predictions of the model correspond with the actual performance of the capital market. We will comment on the relaxation of these assumptions later.

These assumptions describe a frictionless market in which rational economic men and women conduct their business. In this market, investors would indeed hold the market portfolio, as described earlier, and the market would reach equilibrium when all investors reached the market portfolio position. Now how would assets would be priced in such a market?

The rational investor in our assumed world is concerned with the expected return and variance (or standard deviation) of his or her investment portfolio. Any individual security that is purchased will affect the return and risk of the investor's portfolio, and the attractiveness of that asset to the investor therefore depends on these portfolio effects. Since the equilibrium portfolio is the market portfolio, each asset will be priced according to its contribution to the risk and return of the market portfolio. Since the expected return of a portfolio is simply the weighted average of its component securities, an individual asset will increase the portfolio expected return if that asset has a higher expected return than the portfolio. A lower expected return will pull down the portfolio return. However, the incremental risk the asset brings to the portfolio rests not so much on the riskiness of the asset as on its covariance with the portfolio. This leads to the partitioning of risk into *systematic* and *unsystematic risk*.

Consider two securities that have comparable expected returns and, according to past experience, have displayed similar levels of risk. However, the securities have exhibited very different correlations with returns on the market portfolio. As seen in Figure 6(a), security A has tended to deliver high returns during periods when returns on the market portfolio were high and correspondingly low returns when returns on the market portfolio were low. Each dot in the diagram represents the pair of returns for the security r_A, and the market portfolio r_M during a given period. There is not a perfect correlation, which suggests that there is a random, or unexplained, component to the return of stock A. Nevertheless, there is a strong correlation, and the relationship is represented by a straight line. This line is constructed by fitting a simple regression as follows:

(4) $$r_{A,t} = \alpha + \beta\,(r_{M,t}) + e_{A,t}$$

where $r_{A,t}$ = the return on A in period t

$r_{M,t}$ = the return on the market portfolio in period t

α & β = estimated regression coefficients

$e_{A,t}$ = the residual error in period t

The term β shows the sign and sensitivity of the relationship that exists between A and M. In the case illustrated in Figure 6(a), r_A moves upward on average as r_M moves upward; thus β has a positive slope. However, the slope is rather less than 45°, indicating that a percentage movement in r_M would be associated with rather less than a 1% movement in r_A. Thus β would have a value of something less than one. In Figure 6(b), a negative β security is shown. With this security, rates of return have tended to move in the opposite direction to the return on the market portfolio. It is clear that β is really measuring a similar relationship to that represented by the covariance. In fact, β is nothing more than a scaled version of covariance:

$$\beta_A = \frac{cov\ (r_A\ r_M)}{\sigma^2(r_M)}$$

where $cov\ (r_A\ r_M)$ = the covariance between r_A and r_M
$\sigma^2\ (r_M)$ = the variance of the return on the market portfolio

Examination of equation (4) and Figure 6(a) and (b) reveals that the riskiness of the security is divided into two components. Part of the risk is explained by the relationship between the security's returns and the return on the market index. This risk is transmitted through the sensitivity coefficient β (or covariance). Thus, as the market moves up or down, there are sympathetic movements in the individual stock's returns. For stock A, the explained movement is in the same direction as the movement in returns on the market portfolio; for stock B, the movement is in the opposite direction. This explained component of risk is labeled *systematic risk* (otherwise known as *market risk)* because it is systematically related to movement in the market portfolio. The higher β, the more sensitive the stock's returns are to changes in the market portfolio.

The second component of risk also is apparent from examination of equation (4) and Figure 6. The distribution of security returns in Figure 6(a) and 6(b) is not fully explained by movement in the market portfolio. The various dots in the diagrams show some random variation about the correlation line, revealing that there is an unexplained variation in security returns. The unexplained variation in security returns is also shown by the error term $e_{A,t}$ in equation (4), and the unexplained component of risk is known as *unsystematic, or non-market, risk.*[2]

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[2]The partition of risk can also be seen as follows:

$r_A = \alpha + \beta r_{M,t} + e_{A,t}$ from equation (4)

However, the expected return on security A is

$E(r_A) = \alpha + \beta E(r_M)$

Therefore, $\sigma^2(r_A) = E[r_A - E(r_A)]^2$

$= E\ \{[(\alpha + \beta r_{M,t} + e_{A,t}) - [\alpha - \beta E(r_M)]\}^2$

$= E\{\beta\ [r_{M,t} - E(r_M)] + e_{A,t}\}^2$

The Security Market Line and the Capital Asset Pricing Model

The market portfolio is, by definition, highly diversified. Consequently, the unsystematic component of risk will be almost "diversified out". The unsystematic risk of each security will have little or no effect on the overall riskiness of such a diversified portfolio. As a result, the investor should be indifferent to the degree of unsystematic risk exhibited by an individual security, and this will have no effect on the price of the security.

Systematic risk is an entirely different matter. If the covariance with the market portfolio is high (that is, β is high), the addition of the security to the investor's diversified portfolio will have an adverse effect on the portfolio risk. Conversely, if β is low, the addition of the security will have a beneficial effect on portfolio risk. In the somewhat unusual case that the covariance (and therefore, β) is negative, the inclusion of the security can bring a big reduction in portfolio risk. Thus, it is the covariance with the market portfolio, or β, that determines the incremental contribution of each security to the riskiness of the typical investor's portfolio, and consequently, β exerts an important influence on the security price.

The capital asset pricing model (CAPM) embodies a linear relationship, in equilibrium, between the expected return of a security and the security's β value. This relationship is known as the *security market line* (SML) and is shown in Figure 7. The security market line reveals that price is determined by systematic risk alone. Unsystematic risk does not affect price, and thereby does not affect the expected return, because this element of risk is substantially eliminated by simple diversification.

In Figure 7, two stocks are represented that do not lie on the SML. For stock X, the expected return is high relative to its p value. The returns on stock X will be viewed as abnormally high relative to β and it will be viewed as being underpriced. Investors will seek to purchase this security, thereby bidding up its price. The price rise will result in capital gain to existing holders but, for those buying at the higher price, the expected return (based on the new price) will be correspondingly lower. In

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$$= E\{\beta^2 [r_{M,t} - E(r_M)]^2 + 2e_{A,t} [r_{M,t} - E(r_M)]\beta + e_{A,t}^2\}$$
$$= \beta^2 \text{ var } r_M + \text{var } e_A \qquad \text{since } E(e_{A,t}) = 0$$

or $\sigma^2(r_A)$ = systematic risk + unsystematic risk this manner, the expected return will fall until the equilibrium relationship is restored. Conversely, security Y is overpriced according to market expectation. Excessive selling pressure will drive the price down, causing expected return to be restored to its equilibrium level.

Two positions on the SML may be pointed out. The market portfolio must have a value of unity, since β measures the correlation with itself. The expected return on the market portfolio is therefore illustrated at β=1.0. The intercept on the vertical axis shows the expected return for a security that is uncorrelated with the market portfolio. Such a security will have a return of r_f (i.e., the risk free rate), keeping in mind that a risk free security also must have no systematic risk. Comparing these two rates reveals that the market portfolio commands an additional return for systematic risk of:

(5) $$E(r_M) - r_f = \text{market risk premium}$$

Securities with β values in excess of unity will command higher returns, and thus a higher risk premium, than the market portfolio. When β is lower than unity, the risk premium will be less than that for the market portfolio. Some securities may even have negative β values. These securities would be in high demand because of their facility for counteracting portfolio risk. Consequently, expected return would be very low, even lower than the risk-free rate. The risk premium on securities with negative values is negative.

The capital asset pricing model (CAPM) was derived independently in the mid 1960s by Sharpe (1964), Lintner (1965), and Mossin (1966)[3] and represented a natural evolution from Markowitz's portfolio theory of the 1950s. We now present the CAPM more formally to reveal the structure of security returns. The security market line is stated by the following equation:

(6) $$E(r_j) = r_f + \beta_j[E(r_M) - r_f]$$

where r_j = the return on security j

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[3]Though, in fact, Karl Borch had already published in 1962, a model of reinsurance markets which was identical in all relevant respects to the CAPM and which he argued could be applied to capital asset markets. So, it is arguable that Borch should receive credit for the CAPM.

β_j = the β value for security j

The first component of any security's return in equation (6) is simply the riskless interest rate. The second component is the risk premium, which is the β value times the market risk premium. Notice that equation (6) shows the expected return in equilibrium to be a linear function of the security's β value. Consistent with the earlier analysis, the equation does not contain any reward for unsystematic risk because this is irrelevant to the security price.

Relaxing the Assumptions of the Model

Clearly, the assumptions used to develop the capital asset pricing model represent an oversimplification of reality. Again, we emphasize that it is not appropriate to reject the model on the basis of its assumptions; rather, we should test whether its predictions correspond with observed behavior. However, we will illustrate how changes in the assumptions affect the predictions. The treatment is illustrative rather than exhaustive, since this is peripheral to our main purpose and an extensive literature exists on the subject.

One assumption that clearly does not hold up is that there exists a single risk-free interest rate for borrowing and lending. Clearly, intermediaries have transaction costs and wish to make some profit. Therefore, borrowing rates typically are higher than lending rates. This implies that there is not one tangency portfolio on the capital market line, but two; one corresponding to the borrowing rate and one corresponding to the lending rate. Labeling these portfolios M_B and M_L, respectively, yields two expressions for the security market line:

$$E(r_B) = r_{f,B} + \beta[E(r_{M,B}) - r_{f,B}]$$
$$E(r_L) = r_{f,L} + \beta[E(r_{M,L}) - r_{f,L}]$$

where $r_{f,B}$ and $r_{f,L}$ are the respective borrowing and lending rates. These security market lines are shown in Figure 8(a). The result is that there is no unique equilibrium price for each security. For some investors, trade in a given security will be in the context of a lending portfolio; for others, trade will be supported by a borrowing portfolio.

Consider another assumption; that there are no transaction costs. Without this assumption, investors may observe small differences in expected returns (and therefore prices) from their estimated equilibrium values without being induced to trade. Divergence from equilibrium must be sufficient to cover transaction costs before trading occurs. This suggests that the SML may not be a single line but a band, as shown in Figure 8(b). The width of the band will depend on the size of transaction costs: The smaller the transaction costs, the narrower the band. Differences in expectations among investors may also produce a range of expected returns for a given β value, also producing a band rather than a unique security market line. The resulting band is shown in Figure 8(c).

Changes such as these produce a pricing relationship that is not as sharp as that shown in the simple model but still has the same general properties. Thus in Figure 8(d), a fuzzy upward sloping area is shown in place of a well defined equilibrium pricing line. The fuzziness reveals some range of uncertainty in the equilibrium price, but this is insufficient to destroy the main insight of the model; that expected returns in equilibrium bear a positive relationship to systematic risk.

Relaxing the assumptions of a unique borrowing/lending rate and no transaction costs implies that investors may hold different portfolios. However, this does not destroy the valuable insight of the model that much risk reduction can be achieved by diversification. The extent of risk reduction depends on the covariances of the securities in an individual's portfolio. Strictly speaking, the use of β values to measure these covariances is appropriate only if investors actually hold the market portfolio. However, if individuals hold portfolios that are highly correlated with the market portfolio, β serves as a useful proxy for these covariances. Most diversified portfolios are highly correlated with the market portfolio, and consequently, β serves as a useful measure of systematic risk.

Some modifications of the capital asset pricing model (CAPM) have been formulated in response to some of its stronger assumptions. For example, Merton (1973) formulated a multi period asset pricing model, Gonedes (1976) formulated a capital asset pricing model with heterogeneous expectations, and Mayers (1972) formulated a model with some nonmarketable assets. Perhaps one of the more interesting developments has been the introduction by Ross (1974) of the arbitrage pricing model, which, *ceteris paribus,* measures systematic risk in relation to multiple factors. These developments do represent improvements in our understanding of capital markets. However, they have not displaced the CAPM but have qualified and fine tuned it. They all preserve the basic insight of CAPM, that investors diversify and that only risk that is undiversifiable is priced. However, most of the newer models predict that more than one factor explains the variation in asset prices. Rather than having a single beta, many of the derivative models explain asset returns with respect to several macro economic variables. We will now see whether the predictions of the CAPM have empirical support.

TESTING THE CAPITAL ASSET PRICING MODEL

Many tests have been conducted on the capital asset pricing model and its various
derivatives. It is difficult to summarize this literature and to arrive at an unqualified conclusion as to whether the CAPM is supported or rejected by the evidence. I will suggest that the mainstream opinion among academics and practitioners is that, while earlier test of the simple single beta CAPM provided some support, later empirical work favors the more complex derivatives of CAPM in which multiple common factors explain much of the variation in asset returns.

As pointed out by Miller and Scholes (1972) many of the earlier test of CAPM had been plagued by statistical problems. One of the landmark studies designed to overcome these problems is that of Fama and McBeth (1973) who showed that portfolios formed (and reformed) according to the rankings of the stock betas, did indeed exhibit rates of return that were linearly related to the portfolio betas as predicted by the CAPM. Contrary evidence on the single factor CAPM also exists, e.g. Levy, (1978) and, more recently, in a series of studies by Fama and French (1993, 1996a, 1996b) and there is still disagreement as to whether the simple CAPM is "alive or dead". However, even if we accept the negative results on the single factor CAPM, Fama and French are amongst a number of people who have shown that asset returns can be well explained by a small number of common factors, say three to five. A full analysis of these issues is not appropriate here. However, the message that is relevant is as follows. We do not know whether the undiversifiable risk is adequately measured by a single factor (the stock beta) as predicted by the simple CAPM or by a small number of common economic factors as in the derivative versions of the CAPM or the arbitrage pricing theory. But this distinction is not really important for our purposes. Empirical work does lend support to the view that some risk is idiosyncratic to each firm and can be diversified if investors hold a number of assets in their portfolio. This risk is not priced and returns on financial assets should be independent of this diversifiable risk. But other risk is common to most firms and this risk is not diversified away simply by adding more stocks to your portfolio. This undiversifable risk is priced and is reflected in rates of returns on financial assets.

IMPLICATIONS FOR FINANCIAL MANAGEMENT

It is now time to draw attention inward to the decisions made within a firm to see how these are influenced by activity within the capital market. The obvious connection is that financial management is assumed to be directed toward the goal of value maximization, and the value of a firm's securities is determined by investor preferences as expressed in their trading activity. The equilibrium rate of return given by the CAPM for a stock with a given β value is the return required by investors to compensate for the systematic risk of that stock. This required rate of return provides a guide for the firm's capital budgeting decisions.

The expected rate of return delivered by a stock depends on the expected future earnings of the firm. Indeed, the equilibrium price of the stock is otherwise defined as the expected value of future earnings of the firm discounted at the equilibrium rate appropriate to the β value. This price implies that the expected return is at its equilibrium rate. With this in mind, consider a new company that has just been floated with a particular investment program in mind. The criterion commonly used to decide whether such a project should be accepted is that the expected cash flows should yield a positive present value when discounted at an appropriate rate. The appropriate rate is simply the equilibrium rate corresponding to the level of systematic risk in the cash flows. This requires calculation of a β value for the cash flows by estimating their correlation with the market portfolio. If the net present value criterion is satisfied, the firm will yield an expected return to the stockholders that exceeds the equilibrium return. Consequently, the stock of the firm will acquire a value in excess of the capital raised by the original flotation. In this way, value is created in satisfaction of the assumed corporate goal.

The preceding paragraph describes the well known capital budgeting framework. The capital budgeting criteria apply to the decision to float a new firm, but they also can be used to evaluate new projects contemplated by an existing firm. The rules can be applied separately to individual projects as long as the correct risk-adjusted discount rate is used. The expected return and β value for a firm can be represented as the weighted average of the expected returns and β values of the various component activities of the firm, that is,

$$E(r) = \sum_i E(r_i)$$
$$\beta = \sum_i w_i \beta_i$$

where $E(r)$ = the expected return from all the firm's activities

$E(r_i)$ = the expected incremental return on component activity i
β_i = the systematic risk for r_i
w_i = the weighting of activity i

The implication of this weighting procedure is as follows: Consider a firm that is currently offering an expected return of, say, 15% to its shareholders, which is the equilibrium rate given the estimated β of 0.8. The equilibrium is illustrated at position A in Figure 9. A new project is contemplated for which the expected cash flows represent a 20% return on new capital to be raised. The β value for these cash flows is estimated at 1.4, and the equilibrium return for this β value is 18%. Clearly, the expected return on this new project, position B, is above the equilibrium return shown by position D. If the new activity accounts for 50% of total activity (that is, $w_i = 0.5$), the above equations show that the firm should have an expected return and β value of:

$$E(r) = 0.5(0.15) + 0.5(0.2) = 0.175$$
$$\beta = 0.5(0.8) + 0.5(1.4) = 1.1$$

This position, C, is definitely above the equilibrium value shown by position E. This excessive return will cause investors to seek to purchase the stock, thereby bidding up its price. In this way, acceptance of the project will increase the share price in satisfaction of the corporate objective.

The relationship between capital market theory and financial management can be summarized as follows. The equilibrium rate of return is the return required by investors given the level of systematic risk. Since this rate is required by investors to supply capital to the firm, it may be considered to represent the cost of equity capital to the firm. This rate reflects the effect of risk on investors' personal portfolios given that they have the facility for diversification. The rate does contain a risk adjustment, but this only relates to systematic risk. Since the corporate objective is assumed to be value maximization, it follows that any investment project accepted by a firm that delivers an incremental return above its equilibrium rate will add value to the firm's stock. This condition will be satisfied if the estimated incremental cash flows from the project yield a positive net present value when discounted at the equilibrium rate of return relevant to the project's systematic risk. Alternatively, the condition for value creation is satisfied if the internal rate of return exceeds the cost of capital. Notice that the new project is not assessed on the basis of its total risk, since part of that risk will disappear when combined with the risk of other activities of the firm and when combined with the risk of other stocks held by the shareholder in his or her investment portfolio. These implications carry over to our consideration of risk management.

IMPLICATIONS FOR RISK MANAGEMENT

The capital asset pricing model is as interesting in what it does not imply for risk management as in what it does. Let us start with the negative. Suppose a firm has a risky cash flow that is uncorrelated with the market portfolio. In other words, this cash flow has a zero beta. Whether the firm retains or hedges this cash flow should be of little concern to the firm's shareholders. Since investors can diversify zero beta risk in the formation of their personal investment portfolios, then the corporation does them no favor by hedging the risk itself. If a firm hedges, the shareholder's portfolio will face a given risk and return which depends on the weight of that firm in the portfolio and upon the composition of the rest of the portfolio. If the firm does not hedge, the investor can replicate the original portfolio risk and return simply by changing its composition; e.g. by reducing the weight of that security. Shareholder diversification is a substitute for corporate hedging.

Let us take another case, where the risky cash flow is correlated with the market portfolio. Under this condition, will corporate hedging add value? The answer depends on the price of risk. Suppose that all insurance policies (or other hedges) are bought for a premium that reflects the beta of the risky cash flow being hedged. Hedging will certainly affect the risk of the firm, and it will certainly affect the beta of the shareholders' portfolios. But investors will neither be better off nor worse off, since the changes in the portfolio are made at exactly the same terms the investors could have obtained by buying and selling stocks. Shareholders could have duplicated the effects of the corporate hedge by trading shares.

But insurance can be mis-priced. If the firm can transfer risk to another party at a price that does not reflect the risk (i.e., the price of the risky cash flow is its present value when discounted at a risk free rate), there can be real gains and losses from the transfer. For example, suppose insurable losses are high when capital market returns are high. Since losses are a negative cash flow, this reverses the sign of the beta, which will be negative. Such losses have the attractive property that they will stabilize a broad portfolio. Thus, they *should* sold at a price which discounts the expected loss at *less than* the risk free rate to reflect the negative beta. This price will be high since the discount rate is low. Thus, if the firm pays a price which is discounted at the risk free rate, then the price is too low and is a bargain. A firm which buys such mis-priced insurance is adding value; not so much by changing the risk, as by buying a policy at a bargain price. The gain is the difference between the equilibrium price and the actual price. Thus, the capital asset pricing model guides us on what the equilibrium market price should be for hedge products.

But notice how little the capital asset pricing model tells us about why we should hedge. If anything it says that, if hedges are priced to reflect beta, corporate hedging is irrelevant and will not benefit shareholders. All it says is that, if we do hedge, we should look carefully at the price, and that the CAPM provides a benchmark. This is an important point; it warns us against a misleading, but widely held view, about why firms should hedge risk. The idea that firms should hedge because shareholders do not like risk carries little credibility in the CAPM world. Either the risk is diversifiable, and can be controlled by shareholders through their own portfolio management, or the risk is undiversifiable, in which cases it will carry a price which reflects the beta. In the last case, the effect on shareholders could be replicated by shareholders simply by trading stocks. *The CAPM does not tell us why firms should manage risk; rather it tells us something about the prices at which insurance and other hedges should sell.*

Thus, we are badly in need of an explanation of how firms can create value through risk management. This will come in chapter 7. However, it is important to note that the CAPM does not argue against hedging. What the CAPM, and derivative asset pricing models tell us, is that the prices of residual cash flows that filter through to investors, should strike a balance between systematic risk and expected returns. The above argument is that hedging will do little in the risk dimension that will make investors better off. The irony is that the hedging story is more about changes the expected value of returns to shareholders. What we will show in chapter 7 is that, if the underlying cash flows of the firm (not the residual claims of shareholders on these cash flows) are risky, this risk will induce opportunistic behavior, increase taxes or produce other inefficiencies that will lower their expected value. This lowering of expected cash flows is a real cost to shareholders. Thus, the role of risk management is not to change the risk-return balance to investors but to reduce the inefficiencies associated with risky cash flows.

The financial management and risk management implications of the capital asset pricing model that are drawn here apply to firms whose ownership is traded. By contrast, consider a family-owned firm whose owners are not solely concerned with value maximization, but with other objectives, such as maintenance of family control. Furthermore, family members may work in the business, thereby combining their ownership interest and their employment interest. For such firms, total risk, not just systematic risk, may be especially important. Family owners probably have not diversified away the joint risk of ownership and employment in the family business. Under such circumstances, ownership interest cannot be separated from employment interest. Indeed, cross subsidization may arise between these separate interests. The value of ownership claims may not be maximized in order to protect family control and to promote the employment interests of family members. For this firm, financial management and risk management will be differently motivated, and sole concentration on systematic risk is inappropriate. Investment projects and risk management proposals may be assessed according to their impact on the total risk of corporate earnings. Although we will concentrate on the more widely held firm in the remainder of this book, we do admit the occasional example in which concentrated ownership dictates the use of more substantial risk premiums.

CONCLUSION

If the value of a firm's equity is to be used to guide risk management decisions, the process by which the capital market values securities merits examination. For risk management in particular, the examination of capital markets is essential, since risk and the potential for reducing risk through diversification are at the heart of the valuation process.

The capital asset pricing model rests on the propensity for an investor to hold a portfolio. The risk of a security may be divided into systematic and unsystematic components. Systematic risk is that which is explained by the correlation of the security returns with returns on the market portfolio. The unexplained, or residual risk, is labeled unsystematic risk. The systematic risk measures the incremental contribution of a security to the riskiness of the market portfolio. Since investors are predicted to hold the market portfolio, it is this measure of risk that is relevant to the pricing of the security. Unsystematic risk is not relevant for security pricing because it is easily diversified away. This reasoning leads to a model of security pricing that asserts that expected returns are linearly related to the security's β value and are independent of unsystematic risk. This relationship is known as the capital asset pricing model.

The capital asset pricing model is based on several restrictive assumptions, thereby tempting the reader to reject it as being unpractical. However, the main insights of the model do survive relaxation of these assumptions. Furthermore, the acid test of the model is not whether its assumptions are realistic, but whether its predictions are confirmed by real-world behavior. Evidence does lend some support for the model in a modified form in which there is a not a single factor, or beta, but several common economic factors which jointly explain variations in asset returns.

The capital asset pricing model and its variants have important, positive and negative implications for risk management. In assessing the cost of capital to appraise risk management (or any other) risk management projects, the relevant measure of risk is the systematic risk of the project, not its total risk. Throughout the book, we will need to discount future cash flows and we will look to the CAPM either explicitly or implicitly to provide the appropriate cost of capital. The second important implication is a negative one. Reduction of risk for a firm may not by itself add value since shareholders themselves can reduce risk in the management of their personal portfolios. Thus, it is unconvincing to argue that firms should manage risk simply because their owners are risk averse. We are lacking a clear motivation for risk management. As we will see in chapters 7 and 8, we must look beyond asset pricing to how risk affect the transactions costs of a firm, in order to explain how value can be created through risk management.

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FIGURE 1.

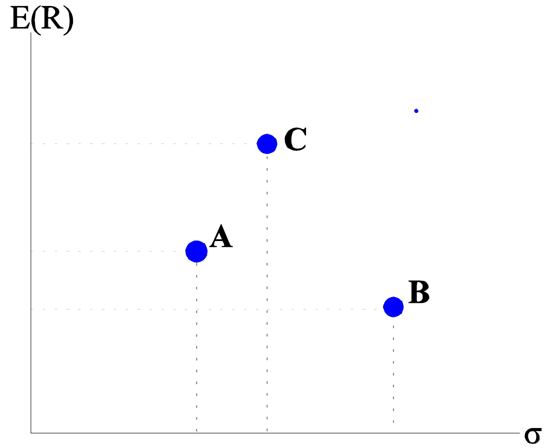
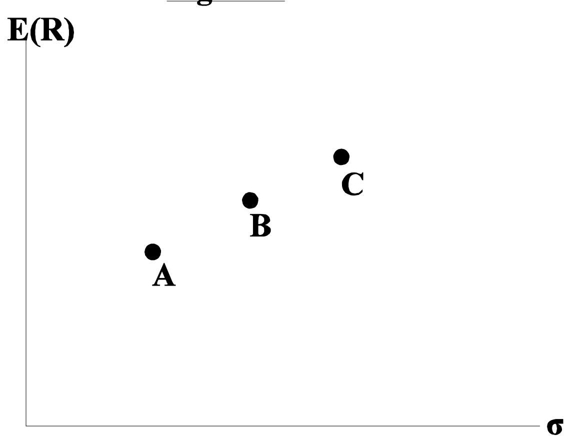


Figure 2



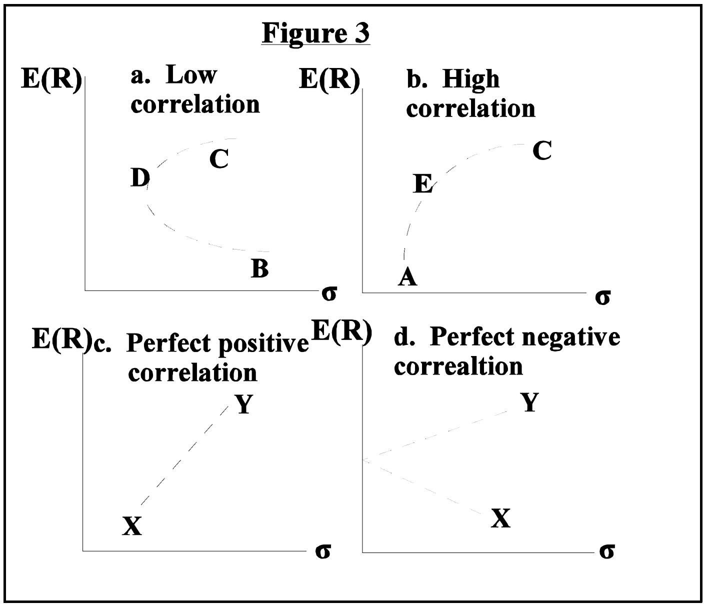


Figure 3
E(R)
a. Low correlation
C
D
B
σ
E(R)
b. High correlation
C
E
A
σ
E(R)
c. Perfect positive correlation
Y
X
σ
E(R)
d. Perfect negative correaltion
Y
X
σ

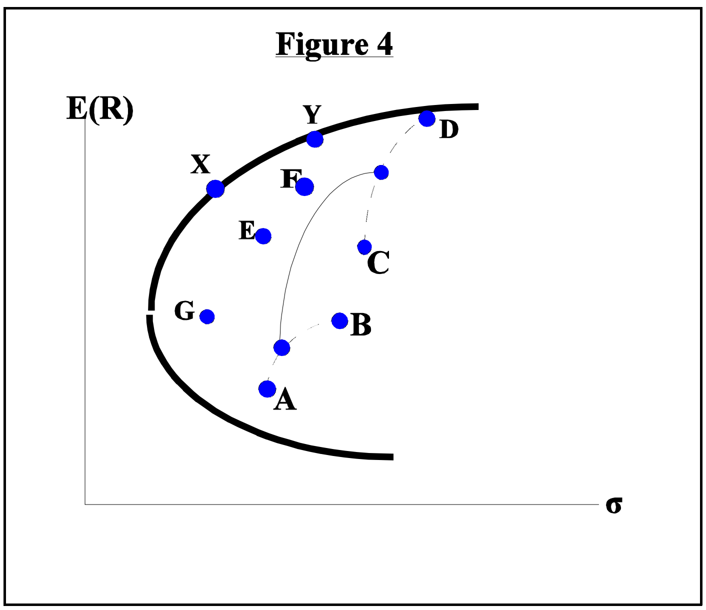


Figure 4
E(R)
Y
D
X
F
E
C
G
B
A
σ

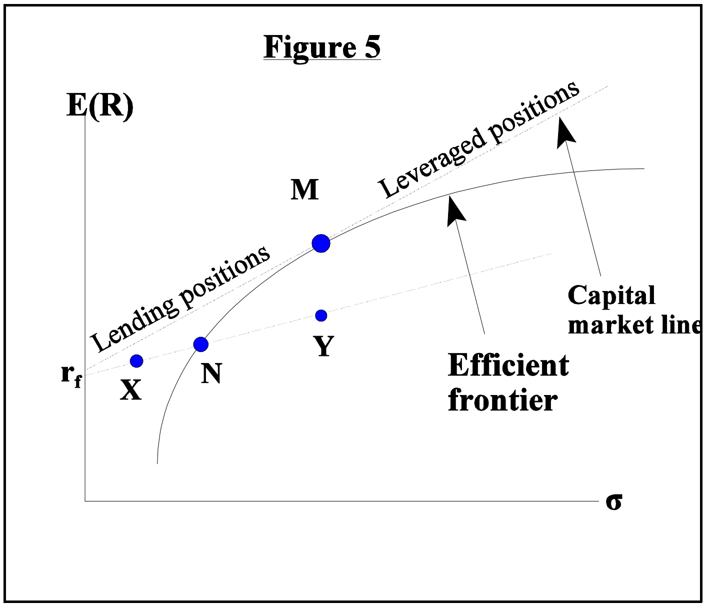


Figure 5
E(R)
Leveraged positions
M
Lending positions
Capital
market line
Y
r_f
X
N
Efficient
frontier
σ

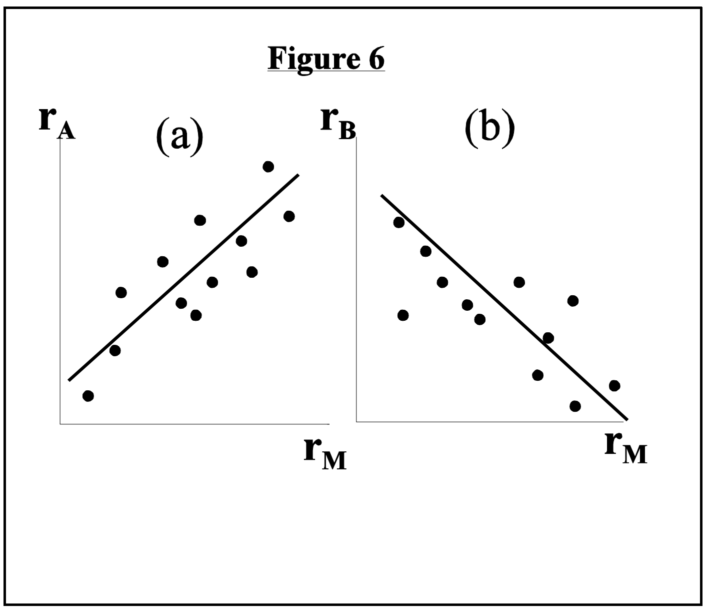


Figure 6
r_A
(a)
r_B
(b)
r_M
r_M

Figure 7

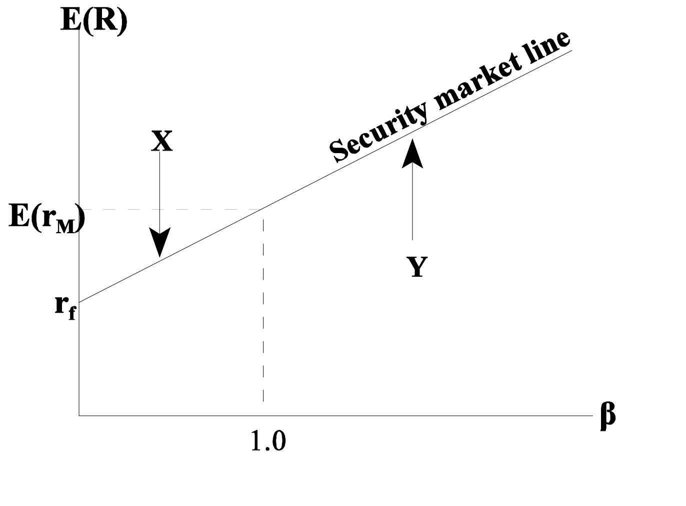


Figure 8

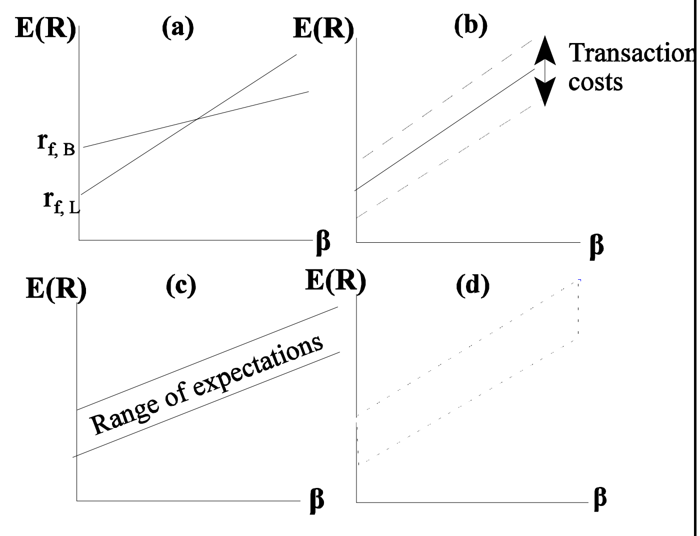


Figure 9