CHAPTER 6, DERIVATIVES AND OPTIONS

INTRODUCTION

In later chapters, we will examine various instruments for managing risk. Amongst the most apparent risk management devices will be hedging tools or insurance. These permit the party holding some risky asset to transfer that risk to another, a counter-party. Risk can be stripped from the asset value, and sold separately for a price. Risk is a traded commodity. The two most familiar hedging products are insurance and options. Insurance has been available as a risk management tool for centuries. Options, though the idea has been around for a long time, have only been used widely in the past couple of decades. Options are one of a class of instruments called derivatives; the class also includes futures, forwards and swaps and various complex combinations of these basic types. All these derivatives have a vital role in risk management and this chapter will serve as an introduction; not to their risk management function but to their basic features.

There is transparency in the naming of derivatives. Unlike Alice's "Wonderland", things are very much what they seem to be. Derivatives are just that, they are instruments that are *derived* from other instruments. Options are what they claim to be, i.e., *choices*; futures and forwards are simply devices for trading some asset in the *future*; and swaps are the *exchange* of one thing for another. This chapter will serve as an introduction to derivatives and as a preparation for some of the uses of derivatives in risk management.

FORWARDS AND FUTURES

Perhaps the simplest derivatives are forwards and futures. These are similar in their overall structure and differ in the details of trade and execution. So let us defer discussion of these differences and start with, say, a forward. For the most part, you are free to purchase or, if you own it, sell any asset. This is the nature of a free market. The price at which you are able to buy or sell the asset will depend upon market conditions. The price for immediate transactions is known as the *spot* price. If, when you wish to sell, spot prices are high, you will do well; but someone else will have to buy at a similar high price. If spot prices were low, you would get a better deal on the buying side and a worse deal on selling. The terms at which you trade are subject to the vagaries of the market; the more volatile the prices, the more you can gain or lose by delaying your transaction. This situation provides opportunities and risks. A buyer facing high prices has the opportunity of waiting in case the spot price falls, and a seller can hold out hoping for spot prices to rise. Figure 1 shows such a possibility. Imagine you are a farmer and you own wheat which you wish to sell. Or a slight variation, it is spring but you will have wheat to sell in the fall when you harvest. If the prices remain as they are in the spring, you will make a given amount of profit. If prices were to rise between spring and fall, you will make more profit. Your graph is the upwards sloping line through the origin. Increases in prices show an increase in profit, and falling prices show a decrease in profit.

Enter the forward contract. As a farmer, the profitability of the coming season will depend on whether prices rise or fall from their present level. The forward contract enables you to lock in to a price. You can agree with a counter-party to sell the wheat in the fall at a price that is fixed today in the spring. The agreement is one to sell an asset at some future date, but at a price to be fixed now (known as the *forward* price). Thus, the contract will be immune from changes in the spot. For the moment, let us say that the forward price is the current spot price. The forward contract will, in fact, yield a profit or loss, depending on which way spot prices change. This is shown in Figure 2. Suppose the farmer buys the forward contract and prices rise by an amount shown as "R". Had the forward not been purchased, the farmer would have been able to sell at the higher price and her profit would have increased by an amount "S". But, the forward contract now means that she has to sell at the forward price and will lose all the profit she would have made had she sold at the higher fall spot price. So a loss, minus "S", is made on the forward contract. The overall position of the farmer is thus to have all profit that would have been made from the price rise wiped out. Why would the farmer do this? Because, in the spring, she does not know whether prices will rise or fall. Suppose prices do fall by an amount "F". Had the future not been in place, the farmer would have lost an amount "G" from selling at the lower fall spot price. Thus, the forward contract, enables the farmer to sell at the current price rather than the lower fall price and thereby to avoid a loss of G. In other words the forward contract makes a profit of "G".

Looking at this from the spring, the farmer will have a profit that is the flat line shown as the horizontal origin. The farmer gives up possible gains from price rises in exchange for avoiding losses from price falls. The same transaction can be seen by the other party. The person buying the wheat forward will lock in the purchase price. Thus, the forward buyer is protected from rising spot prices but sacrifices potential gains from falling spot prices. This can be useful for a baker who will need to purchase wheat anyway. By buying forward, the baker is able to hedge against price increases; just as the farmer is able to hedge against the risk of price falls.

So far we have assumed the forward buyer and seller have some pre-existing interest in the underlying asset and are exposed to risk from future price changes.

This need not be the case. The forward contract can be used purely for a speculative purpose. You may have no interest in the underlying asset, you simply believe that prices are more likely to move in one direction than another. If you believe that prices will fall, then selling the asset forward, will gain you a profit *if* your prediction is correct. You have locked in to the current high price and, if prices fall when you have to deliver (when the contract matures) , you can simply buy at the lower spot price to complete the transaction. If you believe that prices will rise, you can buy forward. If you are correct, you will pay the forward price to complete the transaction which will be higher than the spot price at maturity. If you do not wish to own the commodity, but simply buy the forward for speculation, you can simply resell the good at the spot price and clear the difference between that and the forward price.

Forward Prices

Forward prices are set in relation to spot prices by the following formula. The equilibrium forward price, FW, is:

$$FW = SP(1 + c)^n$$

where SP is the current spot price,, c is the carrying cost and n is the number of years to maturity. The carrying cost, c, requires a little explanation. If you sell forward, rather than now, you will face certain opportunity costs. By holding the asset, you lose the income you could have earned from investing the proceeds of a current sale. For some assets, e.g., commodities, you might have direct costs of storage. On the other side, there can be gains from holding the asset. For example, holding wheat provides an inventory which provides a buffer against supply shortages. The benefits of holding are known as convenience yields. The net carrying cost (after deduction of convenience yields) is the opportunity cost of holding the asset and will be reflected in price. Since this cost is denominated at an annual rate, the total cost is determined by the length of time the asset is held, thus the carrying cost rate is compounded over the “n” periods to maturity. Since the opportunity cost can be positive or negative, the forward price can be either greater or lower than the current spot price.

Forward contracts are not traded on exchanges and are not standardized; they are “over the counter” contracts. These are often undertaken with banks and can be tailor made to match a particular need for hedging or speculation. Forward contracts are common on commodities, currencies and interest rates.

Futures contracts are similar in their underlying structure to forwards. But there are three differences. First, futures are standardized, exchange traded contracts. This makes the contract liquid and will tend to lower transaction costs. Thus, a party holding or writing a future can liquidate his position fairly easily. The second two differences address the credit risk associated with this type of contract. With a forward, there is a risk that the buyer (or the seller) will not be able to perform. Suppose the buyer simply does not have the money to buy the asset at maturity; e.g., the buyer is bankrupt. To guard against this risk, futures contracts adopt first, a device known as "marketing to market" or "cash settled". Each day the balance on the futures contract is settled between the parties. As the futures price changes daily, so the change is settled between the parties. This limits the amount of "debt" between the parties to no more than a single day's change in the forward price and, at maturity, the buyer simply has to come up with the change in the futures price on the final day. The second device is that the buyers and sellers are required to post a performance bond known as a "margin". This is adjusted with cash settlements, but must never fall below an agreed floor. This collateralizes the transaction and minimizes credit risk.

Futures contracts are in effect a sequence of rolling daily forward contracts. Each day a forward contract is closed at the prior day's forward price and a new one opened the current day's forward price. In this sense, the forward contract is a more primitive instrument and becomes a basic building block for more complex derivatives.

OPTIONS

A forward or future is an *obligation* to buy or sell some asset at a future date at an agreed price. An option is a *choice* to buy or sell the asset at an agreed price at some future date. In a forward or future, both parties agree to perform (to buy or sell) as they contracted. In an option, one party, the *option holder* (i.e., with the *long position* in the option) has the choice to buy or sell under the agreed terms. The choice may be exercised or not; there is no obligation. However, the counter-party who *writes* the option (i.e., who has a *short position* in the option), is obligated to sell the underlying asset if the option holder chooses to buy, or buy if the option holder chooses to sell.

There are many types of options; options to buy and options to sell (respectively calls and puts), options with a single exercise time and options in which can be exercised over a time period (respectively European and American), options with cash distributions and options without, etc. We will cover just the basic forms, call and put options and we will illustrate simple European options.

A *call option* is an option to buy an underlying asset. I will illustrate using options on shares of stock. The holder of the call has the option to purchase so many shares of the stock at some future date at a fixed price known as the *exercise price* or *striking price*. The exercise price is denoted "E" here. If the price of the underlying asset, the stock price "M", at maturity exceeds the exercise price, then the person having the long option position, the holder, can exercise the option and buy the stock for E even though it is worth M (>E) at maturity. Thus, the holder makes a gain of M-E. However, the option was purchased up front for some price C and therefore the net profit from the long position in the call is M-E-C. But suppose that M was less than E at maturity. This would mean that, if the option was exercised, the holder would be buying a stock for E when it was worth the lower amount M. The option holder would simply not exercise the option; he would let it expire worthless. In which case the option holder makes zero at maturity and, netting out the cost of the option his net profit is 0 - C which is negative. This profit pattern is illustrated in figure 3 buy the line marked "long call".

The person holding the short position in the call option has no choices to make at maturity, she has sold the choice of purchase of the stock to the option holder. If the holder of the option exercises his choice to buy, the person writing the same option is obligated to honor that choice and sell the stock at the exercise price. With a price rise, the option will be exercised by the holder, and the writer will be forced to sell the stock at E making a negative amount of E-M. But the option was not given away free. The holder had to buy the call option at C and this amount is kept by the writer whether or not the option is exercised. The net profit of the writer, if the option is exercised, is C+E-M. If the stock price at maturity rises only a little, the writer can still be up on the deal since the price will more than offset any small loss on exercise. For a larger rise in the stock price,, the writer will lose money. But, if the stock price is below E, the holder will not exercise and the writer will walk away with a profit of C. Thus, while the holder of a call can make money if the price rises and lose money if the price falls, the profit profile of the writer is exactly the opposite. The profit from the writer of te call option also is shown in Figure 3 as "short call".

The other main class of options is the put option. As a call option is the option to buy an underlying asset, a put option is the option to sell. The option holder, with a long position, buys the right to sell an underlying asset at some future date for a price agreed now. The agreed price is again the exercise, or striking, price. The counter-party, the option writer, sells the right to the put holder, to be able to sell the asset to the option writer. Therefore, if the holder exercises her right to sell at maturity, then the writer is obligated to buy the asset at the exercise price. Figure 4 illustrates the profits to the two parties at different maturity values for the underlying asset. The exercise price is E. If the value of the asset at maturity is above E, the holder will not exercise. There is simply no point is selling the asset at the exercise price E, if the spot price is above E. Therefore the option expires worthless. If the spot price at maturity is below E, then the holder will exercise and clear the difference between the exercise price and the spot. Bearing in mind the holder had to buy the put option for a price P, the net profit for the holder is E-M-P, if $M<E$ and -P if $M \geq E$. Since the profit to the holder is a loss to the writer, and vice versa, the writer's profit is P-E+M, if $M<E$ and P if $M \geq E$.

Hedging and Speculating with Options

Options can be used to hedge a position in an underlying asset. Figure 5 shows the profit profile for a short position in a call option on an asset; together with the value of the asset, denoted "long asset". The value of the asset depends on its price and is therefore shown as the 45° line. Notice that, for values of the asset less than E, the asset value slopes upwards but the call profit is horizontal. Thus, any variation in the asset value in this range will not change the value of the call. But if the asset value finishes in the range above E, any variation in the asset value will be matched by an offsetting variation in the call value; one slopes up at 45° while the other line slopes down at 45°. The combined position is shown by the thick line marked "long asset + short call". The effect of adding the short call to the long asset position is to provide a fixed "bonus" (the call premium) to the asset holder for low values of the asset, but in return the asset holder sacrifices the upside risk. The asset holder is left with most of the downside risk but has hedged the upside.

Figure 6 shows another hedge strategy for the holder of an asset; combining the long asset position with a long put position. Notice now the long put increases in value as the underlying asset falls in value. The combination of these two positions is shown as the thick line. The downside risk from the asset position is hedged, but the upside potential of the asset is retained (minus of course the option premium). This seems a more natural type of hedge; the investor removes the downside risk but preserves upside potential. But options also can be used speculatively. A "naked" option position (holding the option without the underlying asset), means that the holder of the option can make a lot of money if the asset price moves in the right direction, but loses money if the option is not exercised. The call holder is speculating on a price rise and the put holder is speculating on a price fall. The writer also can speculate. Writing a call is speculating on a downward price movement and writing a put is speculating on a upward movement.

Equilibrium Pricing and Put-Call Parity

A call option is said to be “in the money” if the current asset price is above the exercise price and “out of the money” if the current asset price is below the exercise price. Conversely a put option is “in the money” if the current price is below the exercise price and “out of the money” if the current price is above the exercise price. thus, either type of “in the money” option would be profitable to the holder if it could be exercised at the immediate stock price. If the option is “in the money”at maturity, then it is exercised and the holder makes money. If the exercise price is exactly the same as the current price of the asset, the option is said to be “at the money”.

This gives us out first clue about how options are valued and therefore priced. “In the money” options are more valuable than “out of the money” options. Even if the option cannot be exercised immediately, the fact that it is “in the money” now bodes well for it being so at maturity since it does not require a big price change to make a profit. Consider a call option. Given any exercise price, the call is more likely to be “in the money” at maturity, the higher the current price and the lower the exercise price. The value of the call increases as the current price of the asset increases and as the exercise price decreases. These relationships are shown in the following table. The table shows the effect of the asset price on the value of a call option as a “+” and the effect on a put as a “-”; i.e., an increase in the asset price increases the value of the call and reduces the value of the put. Similarly, increases in the exercise price reduce the value of the call and increase the value of the put. There are also other factors affecting the option values and, for our purposes, the most important of these is the volatility of the price of the underlying asset.

FACTORS DETERMINING OPTION PRICES

	CALL	PUT
Price of Underlying Asset	+	-
Volatility or Risk of Asset	+	+
Interest Rate	+	-
Time to Maturity	+	+/-
Exercise Price	-	+

Imagine a call option where the current price of the asset on which is written is 50 and the striking price is 55. If the asset price is perfectly stable, the option will never come "in the money" and is bound to expire worthless. With zero risk to the asset, the call option would have zero value. The same can be said of an "out of the money" put option; with zero risk to the asset, the option will remain "out of the money". Some risk is necessary to give these options value. The greater the risk that is introduced to the asset, the deeper the option can go into the money. Of course, risk works in two directions, up and down. But when looking at options we are only interested in one side of the risk. With a call option, the important thing is the upside risk in the asset. Increasing the downside movement in the asset price will have no effect on the call payoff, any "out of the money" position at maturity yields a zero payoff. But as upside risk increases, this does increase the possible payoffs to the call option. With puts, the reverse is true. Increasing risk to the asset will increase the downside which increases the value of the put and will increase the upside which has no effect on the put value. Thus increasing risk increases the value of both call and put options as shown in the table.

A SIMPLE PRICING MODEL FOR OPTIONS

We will now show a simple ***binomial pricing model*** *to demonstrate how options are priced. The trick in valuing the option is to find "something else" that has exactly the same payoffs as the option. If we know the price of that "something else" then it should sell for the same price as the* *option. If the option were to sell for any different price, then arbitrage would quickly bring the prices together. So, being mysterious about the "something else" let us proceed with an example. The example will simplify the price volatility by assuming the price of the underlying asset can only be one of two values at maturity. Wharton stock[1] currently sells for 50 but it could rise to 77 or fall to 23. Let us find the value of a call option written on Wharton with a striking price of 52 and which can be exercised in one year. First, consider the potential payoffs to the option at maturity.*

	Wharton price = 23	*Wharton price = 77*
Call option value (exercise price = 52)	*0*	*25*

Now consider what the something else might be. Imagine you were to purchase 0.46296 of a share of Wharton stock and borrow 9.68 which had to paid back with 10% interest in exactly one year. This "portfolio" comprising the stock and the debt would have the following value at year end.

	Wharton price = 23	*Wharton price = 77*
0.46296 of share of Wharton stock	*10.648*	*35.648*
Repayment of loan of 9.68 plus 10% interest	*(10.648)*	*(10.648)*
TOTAL	*0*	*25*

The portfolio has exactly the same payoffs as the option and therefore must sell for the same price. Thus the value of the option must be:

Value of Call = hedge ratio (asset price) - present value of loan repayment
= 0.46296 (price Wharton share) - 9.68 loan
= 0.46296(50) - 9.68 = 13.468

Since the payoffs to the option can always be replicated by combining a risk free position (lending or borrowing) with a position in the underlying asset, this formula enables you to value the option. The trick to doing this is to find how much of the underlying asset needs to included in this

[1]Letting Wharton go public is a very interesting, but remote, prospect to contemplate.

portfolio. This proportion is known as the hedge ratio of the option delta (δ). It is calculated as follows;

$$\text{hedge ratio } (\delta) = \frac{\text{spread of option payoffs}}{\text{spread of asset prices}} = \frac{25 - 0}{77 - 23} = 0.46296$$

Let us now value a put option on the same stock where the exercise price is say, 50 (at the money). The payoff at maturity to the holder of the put option is:

	Wharton price = 23	*Wharton price = 77*
Put option value (exercise price = 50)	*27*	*0*

Now the hedge ratio, or δ, is (0-27)/(77-23) = -0.5. But the portfolio positions are reversed. Now you will take a short position in 0.5 shares of Wharton stock and lend 35 at 10% interest. The terminal value of this portfolio is;

	Wharton price = 23	*Wharton price = 77*
short 0.5 of share of Wharton stock	*(11.5)*	*(38.5)*
Repayment of loan of 35 plus 10% interest	*38.5*	*38.5*
TOTAL	*27*	*0*

The portfolio has exactly the same payoffs as the put option and therefore must sell for the same price. Thus the value of the put option must be:

Value of Put = -0.5 (price Wharton share) + 35 loan
= -0.5(50) + 35 = 10

The same general principles of option pricing can be used where the asset prices does not simply fall out as one of two values but can, at maturity of the option, be spread over a very large number of values. The insight is the same, one must establish a portfolio which has the same payoff structure as the option (i.e., for each possible value of the underlying asset, the payoff to the portfolio is exactly the same as the option). This key insight is the meat of the Black Scholes option pricing model. The formula is a little opaque, but the important thing is to note the variables and how they affect value.

$$C = S\,N(d_1) - Ee^{-rt}N(d_2)$$

$$\textit{where}\quad d_1 = \left(\frac{\log(\frac{S}{E} + (r + ½\sigma^2)t)}{\sigma\sqrt{t}}\right) \quad ; \quad d_2 = d_1 - \sigma\sqrt{t}$$

The two terms in the formula for C are essentially the same as those in the binomial formula. Recall that the binomial pricing formula had two terms, the first was the hedge ratio times the current asset price and the second was the present value of a loan. The same is true here. The first term, $SN(d_1)$ is the stock price times the hedge ratio. The second term Ee^{-rt} $N(d_2)$ is really the present value of a loan. The hedge ratio, $N(d_1)$ is rather complicated to calculate since the stock price is changing constantly over time. The expression N(.) is the cumulative normal distribution reflecting that the final value will follow some distribution. The relevant variables are the current price of the underlying asset, "S", the exercise price "E", the discount rate, "r", the time to maturity "t" and the volatility of the asset price "σ". These are all the variables noted in the above table and their effects on price can be worked through the formula to show the signs in the table. The call price increases as the S, σ, r and t increase and the call value decreases as E increases.

An important relationship is that between the price of a call and a put. This relationship is called *put-call parity*. The relationship is an equilibrium one. If prices do not bear this relationship to each other a possibility for arbitrage exists. Therefore, in equilibrium, the prices of call and puts will gravitate to this relationship. Put-call parity asserts the following relationship between the price of a put option and a call option that has the same exercise price.

$$C - P = S - PV(E)$$

The difference between the price of a call option and a put with the same exercise price is equal to the difference between the price of the underlying asset and the present value of the exercise price. In the above example, the value of a put with an exercise price of 50 on a stock with a current value of 50 was 10. Let us now consider what the value of the call would have been had the exercise price been 50 rather than 52. The hedge ratio, or δ, would now be (27-0)/(77-23) = 0.5. And a loan of 10.4545 would be needed to make up the portfolio with the same payoff structure as the call. Thus the call would be priced at

Value of Call = ***hedge ratio (asset price) minus present value of loan repayment***

= ***0.5 (price Wharton share) - 10.4545 loan***

= ***0.5(50) - 10.4545*** = ***14.5455***

So, using put call parity we have

14.5455 - 10 = 50 - 50/(1+0.1) = 4.5455

Had they been priced any differently then money could be made by arbitrage.

SWAPS

Swaps are private agreement in which parties exchange cash flows. The exchange is usually one of different currencies or different interest payments. In the swap, the parties are obligated to the exchange and in this sense, the swap is like a forward or future but not like an option. But, unlike the forward and future, the swap involves the exchange of a number of different cash flows at various future dates. For example, a currency swap might be sought by a U.S. firm with debt raised in Germany for which it must meet D.Mark interest payments. Another firm might be in the opposite position of having to meet an interest obligation denominated in dollars. If these obligations are of roughly equal value (adjusting by exchange rate), then a swap of these interest obligations is feasible. In this way, each firm can replace an obligation denominated in one currency with an obligation in another more suitable currency. Thus, the U.S. firm with some foreign operations, might prefer to have all its obligations denominated in dollars and the swap is a often a convenient and economical way of achieving this.

The other common type of swap is the interest rate swap in which a payment stream of floating rates is exchanged for one in fixed rates. In this type of arrangement, two firms with different needs can agree to exchange the interest payments on some notional principal. For example, one of the firms may have income that is positively correlated with interest rates but its debt obligations are fixed. This firm would be able to hedge its asset risk if it could re-arrange to have its debt interest to be floating. Another firm may find itself with income that is interest insensitive, but has debt obligations that float with interest rates. This firm could avoid the interest risk by replacing its floating interest payments with fixed. So, the two firms have a mutual basis for an exchange of interest payments. More subtly, firms may wish to match the duration of their assets and liabilities and this may call for periodic re-adjustments which can be achieved through the use of swaps.

The needs of firms vary considerably and you can imagine trying to find an appropriate swap partner whose needs exactly mirrored yours. So, swaps are usually arranged with intermediaries who may act as guarantors or who may make a market. This involves a fee for the intermediary to cover the risk in the guarantee and the risk attached to mis-matches in the dealer's overall position.

AN ILLUSTRATION OF AN INTEREST RATE SWAP. A car manufacturer has an expected income stream that has a present value of 100 million. This value is calculated by discounting expected income at the current interest rate of 5%. (i.e., the calculation is 5/0.05 = 100). However, if interest rates change over the next year, this will affect the present value of this cash stream to (values in $ millions):[2]

$$V(A) \quad = \quad \frac{5}{r} \quad = \quad \frac{5}{0.05} + \Delta V(A) \quad = \quad 100 - \frac{5}{r^2}\Delta r$$

The firm also has floating rate debt with a value of $40 m. Note the value of this debt will not change as interest rates change.

$$V(D_1) = \quad r(40)/r \quad = \quad 40$$

In addition, the firm has $40 m million of fixed debt paying a fixed interest of $2 m (5%). Notice that the value of this debt is sensitive to changes in interest rates ($m)

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[2]Note that the change in V(A) in the formula is $-(5/r^2)\Delta r$. This is derived from the total derivative of $dV(.) = \{dV(.)/dr\}\Delta r$; where $V(.) = 5/r$.

$$V(D_2) \quad = \quad \frac{2}{0.05} + \Delta V(D_2) \quad = \quad 40 - \frac{2}{r^2}\,\Delta r$$

The TOTAL FIRM VALUE IS

	$V(A) =$	$100 - (5/r^2)\,\Delta r$
minus	$V(D_1) =$	40
minus	$V(D_2) =$	$40 - (2/r^2)\,\Delta r.$
Net worth		$\mathbf{20 - (3/r^2)\,\Delta r}$

Now suppose that the floating debt of 40m was exchanged for another 40m fixed rate debt. The difference is

	$V(D_1) =$	40
minus	$V(D_2) =$	$40 - (2/r^2)\,\Delta r$
DIFFERENCE		$\mathbf{0 + (2/r^2)\,\Delta r}$

So the firm should be able to buy a swap, exchanging the floating payments on a notional debt of \$40 m for fixed payment of \$2 m for a cost "c" and now assume a liability of $-(2/r^2)\,\Delta r$*. The net worth of the firm is now*

	$V(A) =$	$100 - (5/r^2)\,\Delta r$
minus	$V(D_1) =$	40
minus	$V(D_2) =$	$40 - (2/r^2)\,\Delta r.$
Plus	$SWAP =$	$-c + (2/r^2)\,\Delta r.$
NET WORTH		$\mathbf{20 - c - (1/r^2)\,\Delta r}$

which is much less sensitive to interest rate changes than before the swap. Notice that, if the firm has undertaken the swap on a notional amount of \$60 m for a price of.

$$1.5((-c + (2/r^2)\,\Delta r)$$

Then the firm would have been perfectly hedged

	V(A) =	100	$- (5/r^2)\ \Delta r$
minus	$V(D_1)$=	40	
minus	$V(D_2)$=	40	$- (2/r^2)\ \Delta r$.
Plus	SWAP =	-1.5c	$+ (3/r^2)\ \Delta r$.
NET WORTH			**20m - 1.5c**

You can see now that the hedged value of the firm is independent of "r". The firm has used the swap to remove all risk that arises from changes in interest rates.

SOME APPLICATIONS OF OPTIONS

Debt, Equity and Bankruptcy; the Default Put Option

Debt is a fixed obligation for a firm. In return for borrowing money, the firm commits to a series of interest payments plus repayment of principle. These payments take priority over payments to shareholders; equity is the residual claim. How do we value these claims on the firm? In principle, we can take the capitalized value of the future expected cash flows. Let us simplify this idea in order to show that the financial structure of the firm has clear option characteristics. One of the decisions the firm makes constantly, though often by default, is whether to continue or to liquidate its operations. The decision to continue is rational if the ongoing value exceeds the liquidation value. If this is the case, the firm could be sold for its ongoing value. The sale value is the capitalized value of investors' expectations of future cash flows. This line of thinking enables us to streamline the relationships between debt and equity by compressing the future into a single time period. The present value of the firm is the present value of the maximum of the (a) liquidation value or (b) sale as an ongoing concern value in one period's time. If the firm is liquidated or sold, the value of the equity in one period's time is the residual after accounting for the value of debt.

These relationships are shown in Figure 8. The firm has debt with a face value shown as D. The firm can discharge its debt liability by repaying debt for this amount and the value of the debt is shown as its face value D if the firm is solvent. Suppose the total value of the firm, labeled V(F), is less than D. The firm simply does not have sufficient value to discharge its debt obligation. Under the absolute priority rule of bankruptcy, the equity is now worthless and the whole firm essentially belongs to the creditors. Consequently, the value of debt is the 45° line when $V(F)<D$, and D when $V(F)\geq D$. The value of equity is the residual; zero when $V(F)\leq D$; and V(F)-D when

V(F)>D. The equity value is shown by line coincident with the horizontal axis up to point D and sloping up at 45° thereafter.

The relationships between debt and equity can be seen as option positions. First, notice that the payoff line to equity shown in Figure 8 is the same as that for a call option. If you consider that debt and equity are simply claims on the value of the firm, they are seen immediately as derivatives. Moreover, notice that equity is zero when the underlying asset (the value of the firm) is less than D, and equity is worth the firm value minus D otherwise. Thus, *equity emerges as a call option on the underlying firm value with an exercise price equal to the face value of debt*. This relationship is denoted:

$$V(E) = C\{V(F), D\}$$

This says the value of equity, V(E), is the same as a call option , C{.} on the value of the firm, V(F), where the exercise price is the face value of debt, D. Figure 9 shows the intuition behind thinking of equity as a call option. Start with the 45° line shown as a thin solid line labeled V(F); this is the value of the firm. If we subtract the face value of debt, D, from V(F) we are left with the dotted line, V(F)-D. The downwards arrow shows the effect of deducting D from V(F). Now, also drawn on the diagram is a put option with a striking price equal to the face value of debt. This put option is shown as a dashed line and labeled "*the default put option*" for reasons that will become clear. Now if this put option is added to V(F)-D as shown by the upwards arrow, we are left with the bold solid kinked line which is the equity payoff shown in Figure 8. I.e., we have shown how the equity call option is related to other firm values. These mathematical operations can be shown as follows.

$$V(E) = C\{V(F), D\} = V(F) - D + P\{V(F), D\}$$

where P{V(F), D} denotes a put option written on asset V(F), with an exercise price D. Does this look familiar. Let us re-assemble the relationship as follows:

$$C\{V(F), D\} - P\{V(F), D\} = V(F) - D$$

This is simply put-call parity. Recall, the original notation was C-P = S - PV(E). The right side is simply the difference between the value of a call and put written on the same underlying asset with the same exercise price. The underlying asset here is V(F) (compared with the share price S with stock options) and the exercise price is D. So the only difference is that in the original put-call parity we used the present value of the exercise price on the right side. The reason this was not done in the firm values was simply that we did not think of the time dimension and thus the need for discounting.

Now debt can also be seen in terms of options. We can write the same put-call parity relationship in the following form:

$$V(D) = D - P\{V(F), D\} = V(F) - C\{V(F), D\}$$

This says the value of debt is the difference between the face value of the debt, D, and a put option, P{V(F),D}. Despite the fact that creditors are formally promised a repayment of D, the value of the debt is somewhat less than D; it is D less the value of the default put option. The other side of this coin is that the shareholders have the long position in the default put. They have the right to "put" the firm to the creditors; i.e., the right to exchange the firm for their obligation to pay D to the creditors. It makes sense for the shareholders to exercise this option when the value of the firm is lower than D. So, saying that shareholders have limited liability and that share prices cannot go negative is the same as saying that shareholders own the default out option.

Let us now work through an example to show how the default put option is valued and therefore how debt is valued. To do this we will use the binomial pricing model. It is also useful to re-introduce time and discounting into the model. So let us start by re-stating the put-call parity relationship with the time value of money shown.

$$C\{V(F), D\} - P\{V(F), D\} = V(F) - PV(D)$$

Now, consider the value of a default put option for a firm that is currently worth 132, but may be worth 100 or 200 in one year's time. The firm has outstanding debt with a face value of 120. We will assume a 10% interest rate on riskless debt.

	Terminal value of firm V(F) = 100	*Terminal value of firm V(F) = 200*
Terminal payoff on default put (face value = 120)	*20*	*0*

Now we can calculate the hedge ratio, or δ, using the binomial pricing model considered earlier. The *ratio is that of the spread in the option payoff to the spread in V(F) i.e., (0-20)/(200-100) = -0.2. Now consider that an investor were to take a short equity position in 20% of the firm value and lend 36.36 at 10% interest. The terminal value of this portfolio is;*

	V(F) = 100	*V(F) = 200*
short 0.2 of V(F)	*(20)*	*(40)*
Repayment of loan of 36.36 plus 10% interest	*40*	*40*
TOTAL	*20*	*0*

The portfolio has exactly the same payoffs as the put option and therefore must sell for the same price. Thus the value of the put option must be:

Value of DefaultPut = -0.2 (V(F)) + 36.36 loan
= -0.2(132) + 36.36 = 9.96

The value of the various claims on the firm can now be calculated. Debt is valued as the value of riskless debt (PV(D) = 120 discounted at 10% = 109.09) minus the value of the default put. And equity is the residual claim.

Debt: V(D) = 120/1.1 - 9.96 = 99.13
Equity V(E) = 32.87

And let us finish where we started with put-call parity:

C{V(F), D} - P{V(F), D} = V(F) - PV(D)
32.87 - 9.96 = 132 - 109.09 = 22.91

The default put option is at the heart of the corporate risk management story. The various stakeholders will behave strategically to manipulate the value of the put option to their respective advantages; the shareholders to increase value and the bondholders to reduce it. In seeking advantage at the expense of other stakeholders, shareholders are lulled into decisions that are not value maximizing from the view of the firm as a whole, and the overall level of efficiency of the firm declines. The private, but dysfunctional temptations depend on the level of risk. Thus controlling risk can eliminate this opportunistic game playing and make everyone better off. We shall see in Chapter 7, that much of the rationale of corporate risk management is about jointly controlling risk and leverage and thereby about managing the value of the default put.

The Option to Abandon a Investment Project

The default put option is the option for shareholders to abandon the firm and to pass ownership to creditors. As we have just seen, shareholders will rationally exercise this option when the firm is worth less than the face value of the debt. The same principle applies to individual projects undertaken by the firm. One might suppose that a project is undertaken because its NPV is positive at when the project is first evaluated. But things can change over the project life. At any point in time, the firm can re-assess the value of incremental future cash flows. Of course, the initial capital costs are by-gones. But the remaining life of the project will generate expected cash flows and new estimates can be more or less optimistic than those held at inception. Moreover, there are other potential cash flows that can be gained by closing down the project. The firm can sell off capital equipment, licenses and maybe even goodwill. There may be close-down costs as well as revenues. The firm may have to make severance payments to workers and incur demolition costs. Call the net proceeds of the sale of assets, after deduction of any closedown costs, K^S. Finally, call the the future expected cash flows, C_t , for future year "t". With a discount rate "k", the following formula can be used to determine whether to close down or continue the project.

$$NPV = -K^S + \frac{C_t}{(1+k)^t}$$

The formula shows the NPV from continuing the project, losing the close-down sale value and preserving future cash flows. If this value is negative, the firm will add value by closing down the project. If the value is positive, value will be added by continuing.

To show how the option value plays a part, consider a simple example. A project initially costs 1000 and generates cash flows of 400 per year for three consecutive years. Cash flows are realized at the end of each year. The firm could decide to close down the project immediately after inception and would *recover 900. The firm could also choose to abandon the project and sell off assets at the beginning of year 2 and recover either 700 or 400, each with a 0.5 chance. Similarly, the firm could sell off assets at the beginning of year 3 and realize either 500 or 200 with 0.5 each with chances. The discount rate is 5%. What is the value to the firm from being able to choose to abandon the project at various stages?*

Year	***Initial Capital Cost (beginning of year)***	***Close-down value (beginning of year)***	***Cash Flow (end of year)***
1	*1000*	*900*	*400*
2		*0.5(700) + 0.5(400)*	*400*
3		*0.5(500) + 0.5(200)*	*400*

First, consider the NPV of the project assuming there is no possibility of abandoning it midstream

$$NPV = -1000 + \frac{400}{(1+0.05)} + \frac{400}{(1+0.05)^2} + \frac{400}{(1+0.05)^3} = 89.3$$

Consider what would might happen if the firm closed down at the beginning of year 3. If the firm could close down at this time and recovers 200, it would not do so. Continuing operations into year 3, gives the firm a cash flow of 400 which is worth 400/(1+0.05) = 381 at the beginning of year 3. If the close-down value is 500, at the beginning of the year, then it makes sense to close-down at the beginning of the year rather than wait till year end for 400, (which is worth=381 at the beginning of year 3). So the option to close down at the beginning of year 3 is worth 500-381=119, if the beginning of year close-down value is 500. Given the 0.5 chance of this close-down value, the expected value of the option at inception of the project is 0.5[119/(1+0.5)^2*]= 54. The expected value of the project at the beginning of year 3 is 0.5(500)+0.5(381)=440.5.*

Now look at the beginning of year 2. At this point, the value of continuing the project is:

$$NPV_{year2} = \frac{400}{(1+0.05)} + 0.5\left(\frac{400}{(1+0.05)^2}\right) + 0.5\left(\frac{500}{(1+0.05)}\right) = 800.45$$

Since this exceeds either close-down value,(700 or 400), it is always better to continue with the project at the beginning of year 2. It is also apparent that the firm would not take advantage of closing the *project immediately after inception when the close-down value is 900 (since we know the continuing value is at least 1089.3). Thus, the only potential close-down opportunity which can add value arises at the beginning of year three if the close-down value equals 500.*

Now let us re-evaluate the project accounting for the close-down values.

$$= \quad -1000 + \frac{400}{(1+0.05)} + \frac{400}{(1+0.05)^2} + 0.5\left(\frac{400}{(1+0.05)^3}\right) + 0.5\left(\frac{500}{(1+0.05)^2}\right) \quad =$$

So, the NPV increases from 89.3 to 143.3, an increase of 54, when account is taken of the abandonment option. As a cross check on what is going on, notice also that the value of the abandonment option at project inception is 0.5[119/(1+0.5)^2]= 54 as calculated before.

Abandonment options will play an important role in the risk management story. Consider a couple of examples. First, imagine that the firm is exposed to the possibility of a large product liability suit if it introduces a new product line. If the suit occurs, there is a further chance that, in addition to having to pay damages, the adverse publicity from the suit will dampen future demand for the project. Should this happen, the firm will have the opportunity to abandon the project and sell off project assets. Given that the event has occured and affected demand, the option to abandon cannot make the firm worse and may well help it to limit the extent of the loss. Thus, an important part of the risk management function is how to re-evaluate existing investment projects after a risky event. As a second illustration consider that a firm can lose a productive assets through its physical destruction (e.g., by fire). The fact the asset was in use before the fire, does not imply that its replacement by a new asset will be an automatic positive NPV project. The firm might well choose to use the loss as a reason for abandoning the activity altogether. These issues will be discussed in great detail in later chapters, especially in Chapter 9.

Insurance Policies as Options

An insurance policy can be thought of as a derivative instrument. Consider the value of your home. Now it is worth some amount determined by its condition and by the market for comparable homes. If the condition deteriorates, other things being equal, the value will fall. Now, at the end of the year, the condition might very well have changed; your house may have burned down. Thus, an insurance policy is really like a forward policy on the future value of your house. This needs a little qualification. The policy does not normally cover you for changes brought about by economic trends; rather just changes due to insured perilks such as fire, storm damage, earthquake, etc. Similarly, you can think of an index that records how much you owe to third parties in liability suits. At any point in time, this index is, for most of us, zero. But if we get sued for a traffic accident, the index will suddenly shoot up. An insurance policy is a forward contract on this index where the insurer agrees to assume the index value (a liability) in exchange for an up front premium.

The terms of insurance contracts are often modified to make them look more like options than forwards. For example, a typical car insurance policy will cover losses incurred above a given amount; the deductible. The deductible may be, for example, $500. If losses are less than $500, the policyholder covers the cost; if losses exceed $500, the insurers pays the loss minus $500. The payout structure can be seen in Figure 10 displays the option like feature of the deductible policy. The vertical axis shows how much the insurer will pay based on any loss size recorded on the horizontal axis. The deductible is shown as "D". The insurer pays zero for losses less than D; and pays the loss minus D for losses in excess of D. This payoff structure is shown by the thick line which is zero to the left of D and rises at 45° to the right of D. If we changed the labels, this is exactly the same structure as the payout on a call option with an exercise price of D. And, conveniently, the payment for the insurance, just like the payment for the option, is a "premium".

There is one big difference betwen the insurance policy with a deductible just described and an option such as shares of stock. The latter is an option on another financial asset which is (normally) traded and whose market value can be fairly easily determined. This makes this type of option fairly easy to value and explains the use of the term "derivative". With normal option pricing formulas, the option price is determined in relation to that of the underlying asset; notice that the price of the underlying stock enters the Black Scholes price formula. With an insurance policy, the underlying value on which the option is written, may not be a traded financial asset. The value of a fire loss or a liability suit being examples. This becomes all important for various technical reasons. Suffice it to say here that is would be inappropriate to price the insurance policy using Black Scholes, because the probability distribution that underlies insurance losses, is unlikely to be the same as that which is assumed to specify the movement of stock prices (the lognormal).

CONCLUSION

To many people, the terms "financial risk management" and "derivatives" are inseparable. The rise of financial risk management in the late 1980's and 1990's was largely a movement of using derivatives to hedge interest rate, foreign exchange and commodity risk. But many risk profiles turned out to be complex and simple instruments could not offset the risk. For example, some cash streams were non linearly related to interest rates or commodity prices. Or cash flows could be jointly determined by several exogenous factors. This created a demand for more complex hedge instruments and financial engineers began to combine basic instruments to form new securities. Derivatives, whether used separately, or combined into complex instruments, have become tools of the trade in risk management.

Even for insurable risk, derivatives have become important. Insurance contracts themselves are like derivatives; they can have a "forward like" or an "option-like" structure. Acturial techniques used for pricing insurance are being used to improve the pricing of financial derivatives, and economic techniques used for pricing financial derivatives are being employed in insurance pricing. New types of instuments are being derived which can be used to hedge both insuranble risk and financial risk. And these are being sold both in capital markets and insurance markets. Indeed insurance and derivatives are becoming increasingly interchangeable and their respective market places increasingly integrated.

But our interest in derivatives goes further. As we will see in the next chapter, the reasons that risk is costly to firms is not widely understood. And only recently have credible theories been advanced, and tested, on how the management of risk can enhance corporate value. A big part of this story is related to the potential for risk to bankrupt the firm. This prospect can set the interest of the firm's stakeholders, creditors and shareholders, on a collision course. To understand these conflicts, and how they can be mitigated by the management of risk, it is important to understand the option-like characteristics of the stakeholder claims. In particular, the option of the shareholders to default on their obligations, is the source of the trouble. The management of corporate risk has become, in part, the management of the default put option.

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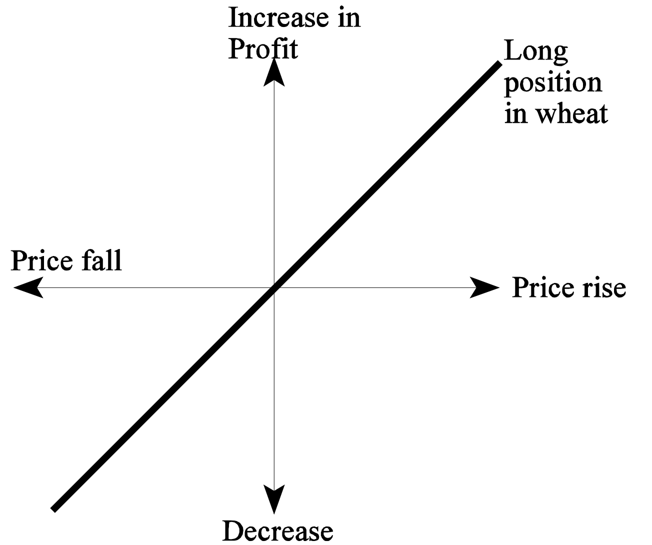


Figure 1. Gain or Loss from Price Change

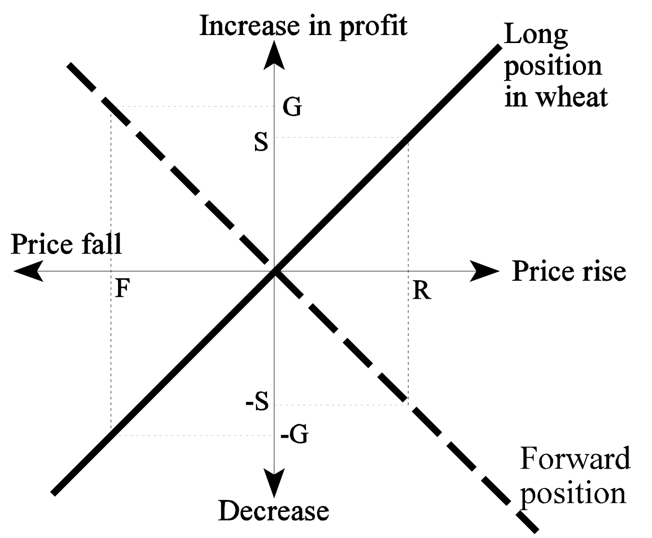


Figure 2. Hedging with Future Contract

Figure 3. Call Options

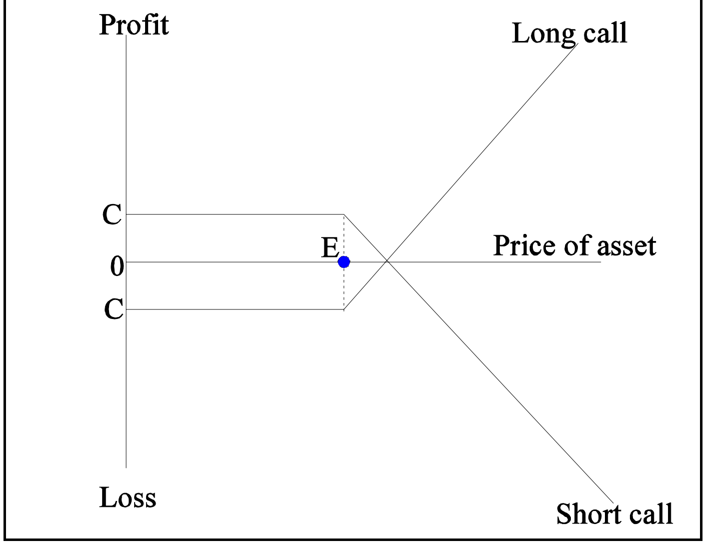
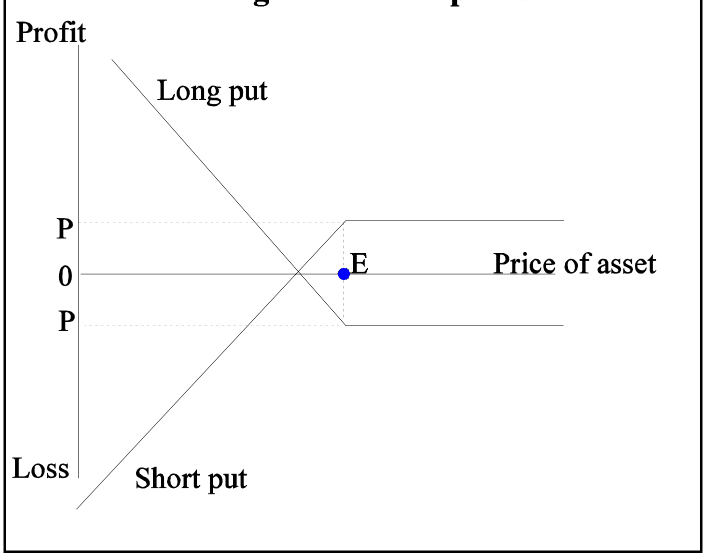


Figure 4. Put Options



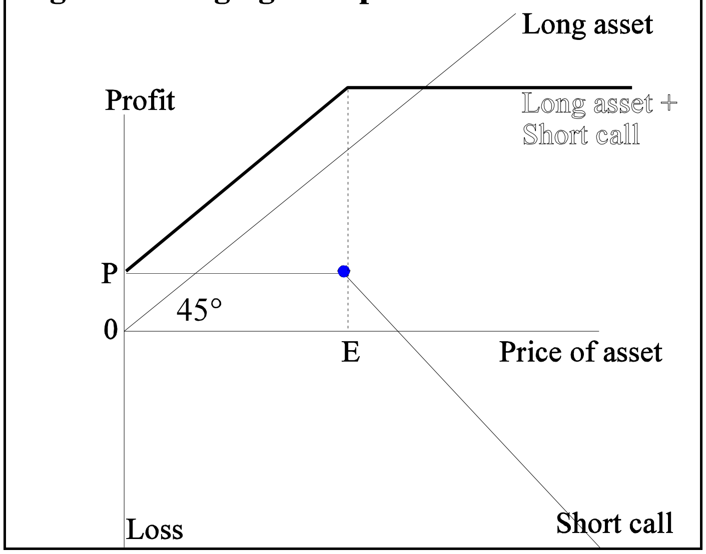


Figure 5. Hedging asset position with short call.

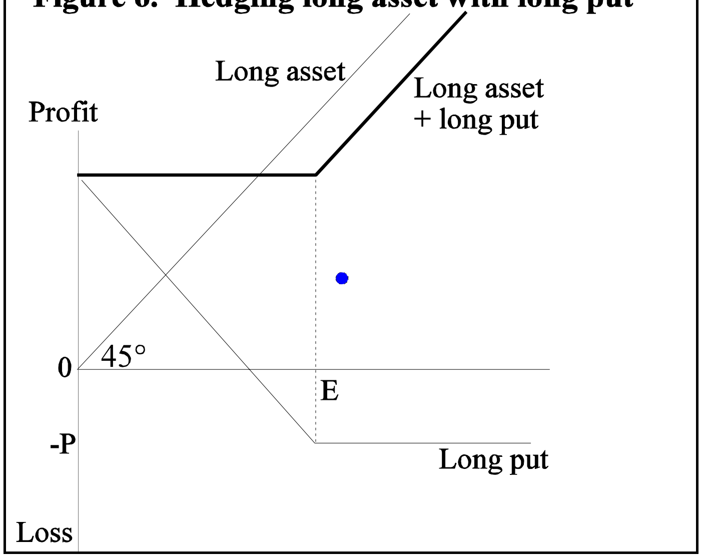


Figure 6. Hedging long asset with long put

Figure 7.
Call option values and current stock prices

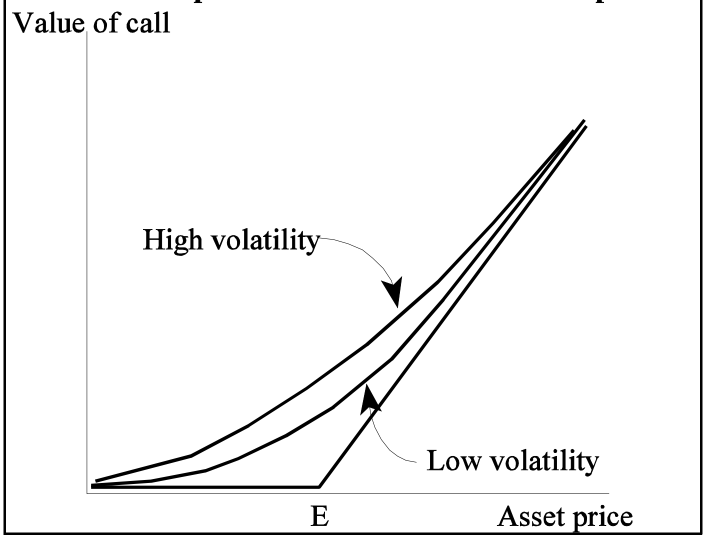


Figure 8. Values of Debt and Equity

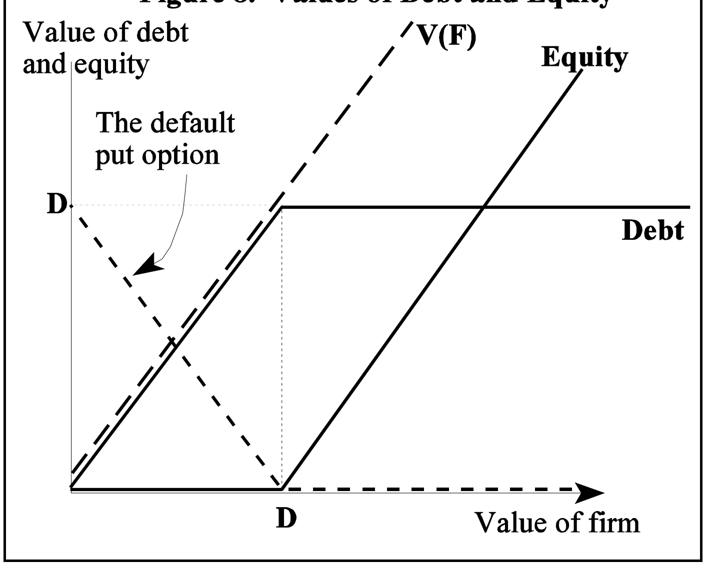


Figure 9. Values of Equity

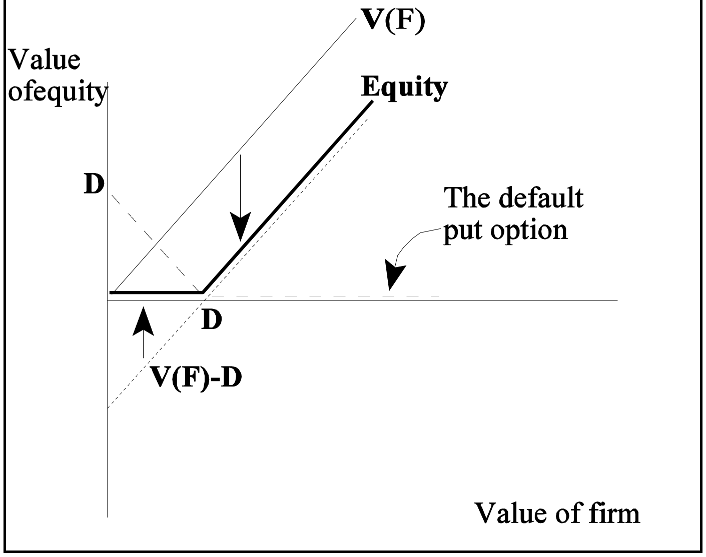


Figure 10. Insurance as and option