Baylor University
Hankamer School of Business
Department of Finance, Insurance & Real Estate

Risk Management
Dr. Garven
Problem Set 3

Name: SOLUTIONS

Problem 1 (40 points). Suppose Moe, Larry and Curley are identical in all respects, except utility. Moe has $U = \ln W$, Larry has $U = 1 + 2W$, and Curley has $U = W^{1.5}$. Moe, Larry and Curley each have initial wealth of \$120 and have a 25 percent probability of losing \$100.

A. Calculate the certainty equivalents of wealth (W_{CE}) for Moe, Larry and Curley.

To answer this question, we must compute the expected utility of this gamble for Moe, Larry and Curley. The state contingent wealths are:

State	p_s	L_s	W_s
Loss	0.25	100.00	20.00
No Loss	0.75	0.00	120.00

Therefore,

$$E(U_{Moe}) = .25 \times (\ln 20) + .75 \times (\ln 120) = 4.34;$$
$$E(U_{Larry}) = .25 \times (1 + 2(20)) + .75 \times (1 + 2(120)) = 191; \text{ and}$$
$$E(U_{Curley}) = .25 \times 20^{1.5} + .75 \times 120^{1.5} = 1,008.26.$$

W_{CE} is computed by setting $E(U(W)) = U(W_{CE})$ and solving for W_{CE}:

$$W_{CE}^{Moe} = e^{4.34} = \$76.67;$$
$$W_{CE}^{Larry} = (191 - 1)/2 = \$95; \text{ and}$$
$$W_{CE}^{Curley} = 1,008.26^{2/3} = \$100.55;$$

B. Who is willing to pay the most to insure this risk? Explain why.

Moe is willing to pay the most to insure this risk; specifically, he is willing to pay *up to* 120-76.67 = \$43.33, which is \$18.33 greater than the actuarially fair value. Since Larry is risk neutral, he is not willing to pay any more than the actuarially fair value of \$25. Finally, since Curley is a risk lover, he is only willing to insure if the price is *less than* actuarially fair value. Specifically, he is not willing to pay any more than \$19.45 to insure this risk.

Problem 2. Suppose you wish to insure an asset valued at \$200. Only two states of the world can occur in the future, FIRE and NO FIRE, with probabilities .2 and .8 respectively. In the FIRE event, the asset is completely destroyed. Your initial wealth (including this asset) is \$300, and your utility $U(W) = \sqrt{W}$.

A. Suppose an insurer offers to fully insure your fire risk for a price of \$40. Should you purchase this insurance policy? Why or why not?

   The fact that it is optimal to purchase this policy can be numerically confirmed by calculating the expected utility of being fully insured and comparing this with the expected utility of being self-insured. By purchasing insurance for \$40, this means that I have a choice between *certain* wealth of \$260 (full insurance case) and a risky lottery with an expected value of \$260 (self-insurance case):

$$\text{Full insurance: } E(U(W)) = .2\sqrt{260} + .8\sqrt{260} = \sqrt{260} = 16.1245, \text{ and}$$
$$\text{Self-insurance: } E(U(W)) = .2\sqrt{100} + .8\sqrt{300} = 15.8564.$$

   This simple numerical example showcases the famous Bernoulli principle, which states that risk averse decision-makers will find it optimal to purchase full coverage if insurance is actuarially fair.

B. If the price for full coverage is \$60, should you full insure? Why or why not?

   By purchasing insurance for \$60, this means that I have a choice between certain wealth of \$240 (full insurance case) and a lottery with an expected value of \$260 (self-insurance case):

$$\text{Full insurance: } E(U(W)) = .2\sqrt{240} + .8\sqrt{240} = \sqrt{240} = 15.4919.$$

   Since full insurance has lower expected utility than self-insurance, I prefer to self-insure.

C. What is the maximum price you are willing to pay to fully insure this risk? Explain how you determined the answer to this question.

   The maximum price is equal to the actuarially fair price plus the risk premium, which is calculated as the difference between expected wealth minus the certainty equivalent of wealth under the self-insurance option. Since 1) my utility is $U(W) = \sqrt{W}$, 2) expected utility of self-insurance is $E(U(W)) = 15.8564$, 3) certainty equivalent of wealth is $W_{CE} = 15.8564^2 = \$251.43$, 4) my risk premium is $E(W) - W_{CE} = \$260 - \$251.43 = \$8.57$, it follows that the maximum price I am willing to pay to fully insure this risk is $\$40 + \$8.57 = \$48.57$.