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Risk Management
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Problem Set 8

Name: SOLUTIONS

Problem 1 (40 points)

Suppose the current value of a (non-dividend-paying) stock is $5,000, and the annual continuously compounded riskless rate of interest is 4%. Based on the example provided on pp. 9-13 from the "Derivatives Theory, Part 1" lecture note, solve parts A and B below.

A. (20 points)What is the "arbitrage-free" price for a forward contract on this stock which matures 1 year from today?

SOLUTION: $F = Se^{rT} = \$5,000e^{.04} = \$5,204.05$

B. (20 points) Suppose the forward price is $5,225. Describe a profitable zero risk, zero net investment trading strategy involving the forward contract and its replicating portfolio. If you implement such a strategy, how much profit will you earn?

SOLUTION: Since the forward price of $5,225 exceeds its "arbitrage-free" price by $20.95, it is too expensive. In order to take advantage of this mispricing in a way that involves zero risk and zero net investment, one should sell a 1-year forward contract for $5,225, buy the stock for $5,000, and fund the stock purchase with $5,000 in (riskless) debt. The following table succinctly summarizes this trading strategy:

Transaction	**Payoff now**	**Payoff @ *T***
Sell Forward	$0	$5,225 - S_T
Buy Stock	($5,000)	S_T
Borrow	$5,000	($5,000)$e^{.04}$ = ($5,204.05)
Arbitrage Profit	**$0**	**$20.95**

Problem 2 (60 points)

The price of a share of Zoom stock is currently $250. It is known that at the end of 1 year, the Zoom share price will be either $312.50 or $200. The riskless interest rate is 3% per year.

A. (10 points) Calculate the price of a 1-year European call option on Zoom stock with an exercise price of $250 by applying the replicating portfolio approach.

SOLUTION: According to the replicating portfolio approach:

$$\begin{aligned}
C_u &= Max(0, S_u - K) = 62.50 \\
C_d &= Max(0, S_d - K) = 0 \\
V_{RP} &= \Delta S + B \\
\Delta S &= \frac{C_u - C_d}{S(u-d)} S = \frac{62.50}{250(.45)} 250 = .5556(250) = 138.89 \\
B &= \frac{uC_d - dC_u}{e^{r\delta t}(u-d)} = \frac{.25(0) - .2(62.50)}{e^{.03(1)}(.45)} = -107.83 \\
\therefore C = V_{RP} &= \Delta S + B = \$138.89 - \$107.83 = \$31.06.
\end{aligned}$$

B. (10 points) Calculate the price of a 1-year European call option on Zoom stock with an exercise price of \$250 by applying the delta hedging approach.

SOLUTION: According to the Delta Hedging Approach:

$$\begin{aligned}
V_H &= C - \Delta S = C - \Delta 250. \\
V_H^u &= V_H^d \Rightarrow 62.50 - \Delta 312.50 = 0 - \Delta 40 \Rightarrow \Delta = .5556. \\
V_H^u &= V_H^d = -111.11 \\
V_H &= C - \Delta 250 = C - 138.89 = -e^{-.03} 111.11 = -.9704(111.11) = -107.83 \\
\therefore C &= \$31.06
\end{aligned}$$

C. (10 points) Calculate the price of a 1-year European call option on Zoom stock with an exercise price of \$250 by applying the risk neutral valuation approach.

SOLUTION: The risk neutral probability of an up move is $q = \frac{e^{r\delta t} - d}{u - d} = \frac{e^{.03} - .8}{1.25 - .8} = .5121$. Since the stock is worth $\$250(1.25) = \$312.50$ at the u node and $\$250(.8) = \$200$ at the d node, this means that the call is only in the money at the u node; specifically, it is worth \$62.50 at that node. Therefore, the price of a one-year call option is

$$C = e^{-r\delta t}[qC_u + (1-q)C_d] = e^{-.02}[.5121(62.50)] = \$31.06.$$

D. (10 points) Calculate the price of a 1-year European put option on Zoom stock with an exercise price of \$250.

SOLUTION: According to the put-call parity equation, $C + Ke^{-r\delta t} = P + S$; therefore, $P = C + Ke^{-r\delta t} - S \Rightarrow P = 31.06 + 250e^{-.03(1)} - 250 = \$23.67$.

E. (20 points) Next, add another 1-year timestep; i.e., it is known that at the end of 2 years, the Zoom share price will be \$390.63, \$250, or \$160. Calculate the price of a 2-year European call option on Zoom stock with an exercise price of \$250. Also calculate the price of a 2-year European put option on Zoom stock with an exercise price of \$250.

SOLUTION: Note that the call option is only in the money at the uu node, where it is worth \$140.63. It is worthless at the ud and dd nodes. Applying risk neutral valuation,

the price of a two-year call option is

$$\begin{aligned} C &= e^{-2r\delta t}[q^2 C_{uu} + 2q(1-q)C_{ud} + (1-q)^2 C_{dd}] \\ &= e^{-.06}[..5121^2(140.63)] = \$34.73. \end{aligned}$$

Regarding the (otherwise identical) 2-year put option, put-call parity indicates that the put is worth \$20.17:

$$P = C + Ke^{-2r\delta t} - S = 34.73 + \$250e^{-.06} - \$250 = \$20.17.$$