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Risk Management
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Problem Set 9

Name: SOLUTIONS

Problem 1. For this problem, the following set of definitions applies:

C = current (European) call option price;
P = current (European) call option price;
S = current price of a non-dividend paying stock (underlying asset for both options);
K = exercise price (common to both options);
r = annualized riskless rate of interest;
T = time (in terms of number of years) to expiration; and
σ = annualized standard deviation of underlying asset's rate of return.

For each of the following scenarios (A through D), calculate the missing variable(s):

Scenario	***C***	***P***	***S***	***K***	***r***	σ	***T***
A	?	?	\$30	\$30	6%	50%	0.25
B	\$2.06	\$2.06	?	\$30	5%	25%	0.50
C	\$0.74	\$9.36	\$25	?	4%	30%	1
D	\$2.60	\$2.16	\$30	\$30	6%	?	0.25

SOLUTIONS:

A. SCENARIO A SOLUTION (6 points): Here, we must find the arbitrage-free prices for the call and put options. We do this by applying the Black-Scholes-Merton option pricing formula for pricing the call option. Once we have determined the arbitrage-free price for the call option, we can determine the corresponding arbitrage-free put option price by applying the put-call parity equation.

Since $c = SN(d_1) - Ke^{-rT}N(d_2)$, the key to calculating the value of the call option is in first calculating d_1 and d_2, then $N(d_1)$ and $N(d_2)$, and then combining these probability measures with the current value of the underlying stock and the present value of the exercise price:

$$d_1 = \frac{\ln(S/K) + (r + .5\sigma^2)T}{\sigma\sqrt{T}} = \frac{\ln(30/30) + (.06 + .5(.25)).25}{.50\sqrt{.25}} = 0.1850.$$

Therefore, $d_2 = d_1 - \sigma\sqrt{T} = 1.850 - .5\sqrt{.25} = -0.0650$. Consequently, $N(d_1) = 57.34\%$, $N(d_2) = 47.41\%$, and the value of the call option is:

$$c = SN(d_1) - e^{-rT}KN(d_2) = \$30(57.34\%) - e^{-.06(.25)}(\$30)(\ 47.41\%) = \$3.19.$$

According to the put-call parity equation, $c + Ke^{-rT} = p + S$; therefore, $p = \$3.19 + e^{-.06(.25)}(\$30) - \$30 = \$2.74$.

B. SCENARIO B SOLUTION (6 points): Here, we must find the current market value of the stock. We do this by applying the put-call parity equation; since $c + Ke^{-rT} = p + S$, it follows that $S = c + Ke^{-rT} - p = 2.06 + e^{-.05(.5)}(\$30)$ - \$2.06 = \$29.26.

C. SCENARIO C SOLUTION (6 points): Here, we must find the exercise price. We do this by applying the put-call parity equation; since $c + Ke^{-rT} = p + S$, it follows that $K = e^{rT}(p + S - c) = e^{.04}(\$9.36 + \$25 - \$0.74) = \$35$.

D. SCENARIO D SOLUTION (6 points): Here, we are asked to calculate the standard deviation, which requires computation by trial and error. Since 1) all parameter values for Scenarios A and D are the same except for option prices and volatility, and 2) Scenario D option prices are lower than Scenario A option prices, this implies that volatility must also be lower under Scenario D compared with Scenario A. A precise calculation involving the use of Excel's Solver reveals a volatility measure of 40%. A simple linear approximation which involves setting the call option price ratio (2.60/3.19) equal to the corresponding ratio of standard deviations which produce these prices ($\sigma/.5$) marginally overestimates Scenario D volatility; i.e., $2.60/3.19 = \sigma/.5 \Rightarrow \sigma \cong .5(2.60/3.19) = 40.75\%$.

Problem 2.

Suppose you are interested in determining arbitrage-free prices for a European call option and (an otherwise identical) European put option. The underlying stock does not pay dividends, and its current price is $S = \$100$. For both options, the exercise price $K = \$115$, $u = e^{\sigma\sqrt{\delta t}}$, $d = e^{-\sigma\sqrt{\delta t}}$, and the length of each timestep is $\delta t = 1/6$. Furthermore, the riskless rate of interest $r = 4\%$ per year, the underlying stock's volatility $\sigma = 20\%$ per year, and both options expire 6 months from today.

A. (8 points) What is the arbitrage-free price for the call option?

SOLUTION: Rather than calculate all possible node-specific prices for the underlying stock, it is simpler to determine the minimum number of up moves required in order for the call option to expire in the money. Let $a =$ the smallest integer value $> \ln(K/Sd^n)/\ln(u/d)$. Since $\sigma = 20\%$ and $\delta t = 1/6$, $u = e^{\sigma\sqrt{\delta t}} = e^{.20\sqrt{1/6}} = 1.0851$ and $d = .9216$, it follows that $\ln(115/100.9216^3)/\ln(1.0851/.9216) = 2.356$. Consequently, the call option will only be in the money at node uuu. Since node uuu stock price is $u^3 S = 1.0851^3(100) = \$127.76$ and the risk neutral probability of an up move is $q = \frac{e^{r\delta t} - d}{u - d} = \frac{e^{.04/6} - .9216}{1.0851 - .9216} = .5205$, it follows that $c_{uuu} = \$12.76$ and $c = e^{-.04/2}.5205^3(12.76) = \$1.76$.

B. (8 points) What is the arbitrage-free price for the put option?

SOLUTION: Applying the put-call parity equation, we find that $c + Ke^{-rn\delta t} = p + S \Rightarrow p = \$1.76 + \$115e^{-.04/2} - 100 = \$14.49$.

C. (8 points) Suppose that a change in the company's corporate risk management policy increases annualized volatility from $\sigma = 20\%$ to $\sigma = 30\%$. Calculate the effect that this change will have on the arbitrage-free prices for the call and put options, and explain why these prices changed in the manner that they did.

SOLUTION: The call option increases in value from \$1.76 to \$3.53, whereas the put option increases in value from \$14.49 to \$16.25. Since call and put options are both positively related to changes in volatility, it follows that the arbitrage-free call and put prices must both increase in value when volatility increases.