Finance 4335 Risk Pooling Class Problem Solutions

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1. Suppose insurable losses are normally distributed, and your loss distribution has an expected value of $1,000 and a standard deviation of $1,000. Calculate the probability of personally incurring a loss which exceeds $1,500.

   SOLUTION: Since $1,500 is half of a standard deviation above the expected value of loss (i.e., $z = \frac{L-\mu}{\sigma} = \frac{1,500-1,000}{1,000} = .5$), it follows that the probability of personally incurring a loss in excess of $1,500 is 1- $Pr(z = .5)$ = 1 - .6915 = .3085.

2. Consider an insurer which has sold 5 *independent* and *identically* distributed insurance policies, each having an expected loss of $1,000 and a standard deviation of $1,000. Calculate 1) the expected value and standard deviation of the insurer's average loss distribution, and 2) the probability that the loss on an average policy will exceed $1,500.

   SOLUTION:

   - $E(L_p) = \sum_{i=1}^{n} w_i E(L_i) = (1/n)n\mu = \mu = \$1000$, and $\sigma_{L_p} = \sqrt{\frac{\sigma^2}{n} + \frac{(n-1)}{n}\rho\sigma^2} = \$447.21$.
   - Since $1,500 is 1.118 standard deviations greater than μ = $1,000 (i.e., $z = \frac{L-\mu}{\sigma} = \frac{1,500-1,000}{447.21} = 1.118$), it follows that the probability that the loss on an average policy will exceed $1,500 is 1- $Pr(z = 1.118)$ = 1 - .8682 = .1318.

3. Suppose the insurer sells 10 such policies rather than only 5 policies. Perform the same calculations as in Problem 2 and explain the difference in your results.

   SOLUTION:

   - $E(L_p) = \sum_{i=1}^{n} w_i E(L_i) = (1/n)n\mu = \mu = \$1000$, and $\sigma_{L_p} = \sqrt{\frac{\sigma^2}{n} + \frac{(n-1)}{n}\rho\sigma^2} = \$316.23$.
   - Since $1,500 is 1.5881 standard deviations greater than μ = $1,000 (i.e., $z = \frac{L-\mu}{\sigma} = \frac{1,500-1,000}{316.23} = 1.5881$), it follows that the probability that the loss on an average policy will exceed $1,500 is 1- $Pr(z = 1.5881)$ = 1 - .9431 = .0569.

   DISCUSSION: Since losses are identically distributed, increasing the size of the risk pool has no effect upon the expected value of the insurer's average loss distribution. However, the standard deviation is reduced due to diversification. Since the standard deviation of the average policy is smaller than before, the probability that losses on average exceed a large amount is also reduced.

4. Suppose the insurer sells 10 policies but now believes that the correlation between policies is .1 rather than zero. Compare your results with those obtained in Problem 3 and explain any differences.

SOLUTION:

- $E(L_p) = \sum_{i=1}^{n} w_i E(L_i) = (1/n)n\mu = \mu = \$1000$, and $\sigma_{L_p} = \sqrt{\frac{\sigma^2}{n} + \frac{(n-1)}{n}\rho\sigma^2} = \$435.89.$
- Since \$1,500 is 1.147 standard deviations greater than $\mu = \$1{,}000$ (i.e., $z = \frac{L-\mu}{\sigma} = \frac{1,500 - 1,000}{435.89} = 1.147$), it follows that the probability that the loss on an average policy will exceed \$1,500 is 1- $Pr(z = 1.147) = 1$ - .8743 = .1257.

DISCUSSION: Holding n constant, increasing ρ has no effect upon the expected value of the insurer's average loss distribution. However, for a given n, positively correlated risks are on average riskier than uncorrelated risks. In this case, increasing ρ from 0 to .1 causes the standard deviation of the average loss distribution to increase from \$316.23 to \$435.89. Consequently, the risk of extreme outcomes is greater compared with the previous case (where risks are statistically independent).