Insurance Economics

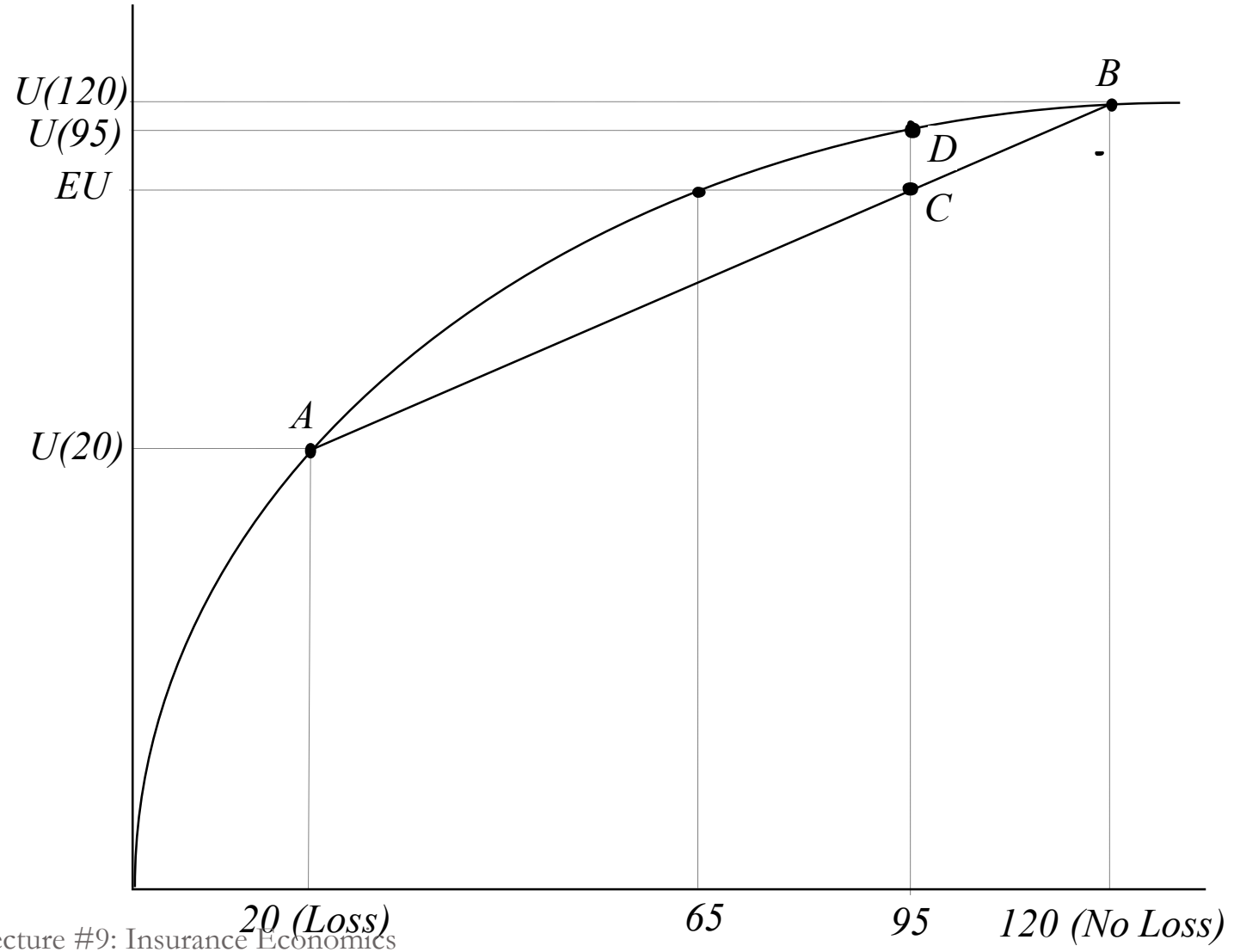
Demand for Insurance

- Bernoulli Principle
- Mossin's Theorem
- Arrow's Theorem

Demand for Insurance

- Suppose you wish to insure an asset valued at \$100. Only two states of the world can occur in the future, FIRE and NO FIRE, with probabilities .25 and .75 respectively. In the FIRE event, the asset is completely destroyed. Your total wealth (including this asset) is \$120, and you are risk averse with utility $U = U(W)$. Using the figure on the following page, answer the following questions:
  - If full insurance costs \$25, should you fully insure? What if the premium is \$35?
  - What is the maximum premium you are willing to pay to fully insure this risk?

Demand for Insurance



Important Insurance Theorems

- Mossin's Theorem: If proportional insurance is available for an actuarially fair (unfair) premium, then full (partial) coverage is optimal (**note:** the Bernoulli principle is a "special case" of Mossin's Theorem).
- Arrow's Theorem: Other things (i.e., premium and expected indemnity value) equal, risk-averse agents prefer insurance policies with deductibles over all other contract forms.

Mossin's Theorem

- Suppose initial wealth (W_0) is \$120, \$100 of which is invested in an asset that has a 25% probability of being destroyed by fire. $U(W) = W^{.5}$, and the premium for a "full coverage" insurance policy is \$25.
- 25% of the time, $W_s = W_0 - \alpha P^i - (1-\alpha)L$, where α is the coinsurance rate and P^i is the price of full insurance coverage. Thus $W_s = 120 - \alpha 25 - (1-\alpha)100 = 20 + 75\alpha$.
- 75% of the time, $W_s = W_0 - \alpha P^i = 120 - \alpha 25$.

Mossin's Theorem

Expected utility is $E(U(W)) = .25(20 + 75\alpha)^{.5} + .75(120 - \alpha 25)^{.5}$. The optimal value for α maximizes expected utility; therefore,

$$\frac{dE(U(W))}{d\alpha} = 9.375(20+75\alpha)^{-.5} - 9.375(120-25\alpha)^{-.5} = 0.$$

$$\therefore (20+75\alpha)^{-.5} = (120-25\alpha)^{-.5}$$

$$\therefore 20+75\alpha = 120-25\alpha$$

$$\therefore 100\alpha = 100 \rightarrow \alpha = 1.$$

In other words, full coverage is optimal when insurance is actuarially fair.

Mossin's Theorem

- Next, suppose that the price of a full coverage policy is \$40. Calculate the optimal value for α.

This price change implies that $E(U(W)) = .25(20 + 60\alpha)^{.5} + .75(120 - \alpha 40)^{.5}$; therefore,

$$\frac{dE(U(W))}{d\alpha} = 7.5(20+60\alpha)^{-.5} - 15(120-40\alpha)^{-.5} = 0.$$

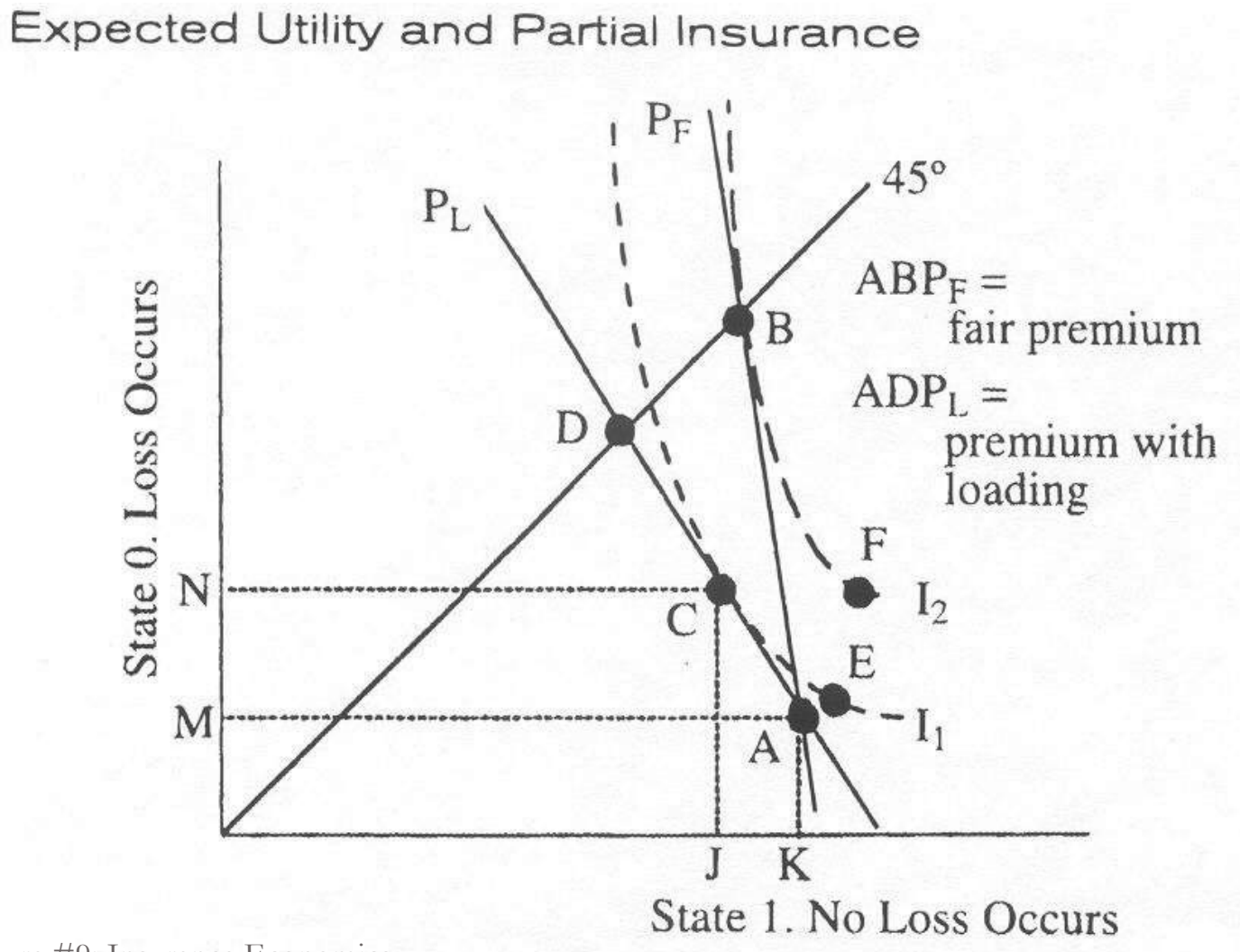
$$\therefore 7.5(120-40\alpha)^{.5} = 15(20+60\alpha)^{.5}$$

$$\therefore 56.25(120-40\alpha) = 225(20+60\alpha)$$

$$\therefore 6,750 - 2,250\alpha = 4,500 + 13,500\alpha$$

$$\therefore 2,250 = 15,750\alpha \rightarrow \alpha = 1/7.$$

Expected Utility and Partial Insurance



Common types of "partial" insurance

- Deductibles – Deductibles limit the loss suffered by the insured to a fixed dollar amount.
- Coinsurance – insurer agrees to pay a fixed proportion of all losses.
- Upper limit – insured retains high severity losses.

Indemnity Schedules Under Different Contracts

1. Full Coverage Contract: $I_s = L_s$; client transfers entire loss to the insurer.

2. Partial Coverage Contracts:

   (a) Coinsurance: $I_s = \alpha L_s$, where α is the coinsurance rate. Under coinsurance, net exposure to loss is

   $$L_s - I_s = (1 - \alpha)L_s.$$

   (b) Deductible: $I_s = \max(0, L_s - d)$, where d is the insurance deductible. Under deductible insurance, net exposure to loss is $L_s - I_s = L_s - \max(0, L_s - d)$.

   (c) Upper Limit: Net exposure to loss under an upper limit contract $L_s - I_s = L_s - \min(U, L_s)$, where U represents the contract's upper limit.

Partial Insurance Numerical Example (1 of 5)

Initial Wealth	$260					
Premium Loading	20%					
U(W) = W^n,						
where n=	0.50					
			Insurance Payment Calculations			
						Upper
		Self	Full	Deductible	Coinsurance	Limit
p(s)	L(s)	Insurance	Insurance	$20	75%	$100
50%	$0	$0	$0	$0	$0	$0
10%	$20	$0	$20	$0	$15	$20
20%	$40	$0	$40	$20	$30	$40
10%	$100	$0	$100	$80	$75	$100
10%	$200	$0	$200	$180	$150	$100
E(.)	$40	$0	$40	$30	$30	$30
Premium		$0	$48	$36	$36	$36

Partial Insurance Numerical Example (2 of 5)

Initial Wealth	$260					
Premium Loading	20%					
U(W) = W^n,						
where n=	0.50					
			Final Wealth Calculations			
						Upper
		Self	Full	Deductible	Coinsurance	Limit
p(s)	L(s)	Insurance	Insurance	$20	75%	$100
50%	$0	$260	$212	$224	$224	$224
10%	$20	$240	$212	$204	$219	$224
20%	$40	$220	$212	$204	$214	$224
10%	$100	$160	$212	$204	$199	$224
10%	$200	$60	$212	$204	$174	$124
E(.)	$40	$220	$212	$214	$214	$214
σ	61.32	61.32	0.00	10.00	15.33	30.00

Partial Insurance Numerical Example (3 of 5)

Initial Wealth	$260					
Premium Loading	20%					
U(W) = W^n,						
where n=	0.50					
			Expected Utility Calculations			
						Upper
		Self	Full	Deductible	Coinsurance	Limit
p(s)	L(s)	Insurance	Insurance	$20	75%	$100
0.50	0.00	16.1245	14.5602	14.9666	14.9666	14.9666
0.10	20.00	15.4919	14.5602	14.2829	14.7986	14.9666
0.20	40.00	14.8324	14.5602	14.2829	14.6287	14.9666
0.10	100.00	12.6491	14.5602	14.2829	14.1067	14.9666
0.10	200.00	7.7460	14.5602	14.2829	13.1909	11.1355
Expected Utility		14.6174	14.5602	14.6247	14.6187	14.5835

Partial Insurance Numerical Example (4 of 5)

Initial Wealth	$260					
Premium Loading	50%					
U(W) = W^n,						
where n=	0.50					
			Final Wealth Calculations			
						Upper
		Self	Full	Deductible	Coinsurance	Limit
p(s)	L(s)	Insurance	Insurance	$20	75%	$100
50%	$0	$260	$200	$215	$215	$215
10%	$20	$240	$200	$195	$210	$215
20%	$40	$220	$200	$195	$205	$215
10%	$100	$160	$200	$195	$190	$215
10%	$200	$60	$200	$195	$165	$115
E(.)	$40	$220	$200	$205	$205	$205

Partial Insurance Numerical Example (5 of 5)

Initial Wealth	$260					
Premium Loading	50%					
U(W) = W^n,						
where n=	0.50					
			Expected Utility Calculations			
						Upper
		Self	Full	Deductible	Coinsurance	Limit
p(s)	L(s)	Insurance	Insurance	$20	75%	$100
0.50	0.00	16.1245	14.1421	14.6629	14.6629	14.6629
0.10	20.00	15.4919	14.1421	13.9642	14.4914	14.6629
0.20	40.00	14.8324	14.1421	13.9642	14.3178	14.6629
0.10	100.00	12.6491	14.1421	13.9642	13.7840	14.6629
0.10	200.00	7.7460	14.1421	13.9642	12.8452	10.7238
Expected Utility		14.6174	14.1421	14.3136	14.3071	14.2690