Baylor University
Hankamer School of Business
Department of Finance, Insurance & Real Estate

Risk Management
Dr. Garven
Problem Set 7

Name: SOLUTIONS

Problem 1.

Suppose that you are interested in investing in a portfolio consisting of stocks and bonds. You have already decided to invest in an equity fund that is indexed to the S&P 500, but are not sure whether to invest in Government bonds or Catastrophe ("Cat") bonds. The Cat bonds are riskier than Government bonds because Cat bond investors must forfeit interest and principal payments if a Category 4 or 5 hurricane hits Florida. The following table summarizes the expected returns and standard deviations for these securities:

Security	Expected Return	Standard Deviation
Cat Bonds	8%	15%
Equity Fund	12%	20%
Gov't Bonds	5%	8%

The correlation between Cat Bond returns and Equity Fund returns is -.1, whereas the correlation between Government Bond returns and Equity Fund returns is .10.

A. Suppose that you would like to earn 10% in your portfolio. What is the standard deviation of a portfolio consisting of Cat Bonds and the Equity Fund that would have an expected return of 10%?

SOLUTION: The expected return on a portfolio consisting of two assets is $E(r_p) = w_1E(r_1) + w_2E(r_2) = w_1E(r_1) + (1 - w_1)E(r_2)$. The expected return on the Cat Bond, $E(r_1) = 8\%$, and the expected return on the Equity Fund, $E(r_2) = 12\%$. Therefore, the expected return on a portfolio consisting of the Cat Bond and Equity fund, $E(r_{CBEF}) = w_1(.08) + (1 - w_1)(.12) = 10\% \Rightarrow w_1 = w_2 = 50\%$, and

$$\begin{aligned}\sigma_{CBEF} &= \sqrt{w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1w_2\sigma_{12}} \\ &= \sqrt{.50^2\left(.15^2\right) + .50^2\left(.20^2\right) + 2\left(.5\right)\left(.5\right)\left(-.1\right)\left(.15\right)\left(.2\right)} = 11.88\%.\end{aligned}$$

B. What is the standard deviation of a portfolio consisting of Government Bonds and the Equity Fund that would have an expected return of 10%?

SOLUTION: The expected return on the Government Bond, $E(r_1) = 5\%$, and the expected return on the Equity Fund, $E(r_2) = 12\%$. Therefore, Therefore, the expected return on a portfolio consisting of the Government Bond and Equity Fund, $E(r_{GBEF}) = w_1(.05) + (1 - w_1)(.12) = 10\% \Rightarrow w_1 = 2/7 \Rightarrow w_2 = 5/7$, and

$$\begin{aligned} \sigma_{GBEF} &= \sqrt{w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1w_2\sigma_{12}} \\ &= \sqrt{(2/7)^2\left(.08^2\right) + (5/7)^2\left(.20^2\right) + 2\,(2/7)\,(5/7)\,(.10)\,(.08)\,(.2)} = 14.69\%. \end{aligned}$$

C. Suppose the riskless rate of interest is 3%, and you can borrow or lend against either of the portfolio combinations described in parts A and B above. Which of these two portfolio combinations is preferred by arbitrarily risk averse investors? Justify your answer.

<u>SOLUTION</u>: The optimal portfolio combination is the Cat Bond/Equity Fund (CBEF) portfolio described in part A of this problem. The reason why this is optimal is that the slope of the capital market line that is generated from lending and borrowing against this portfolio is steeper than the slope of the capital market line that is generated from lending and borrowing against the Government Bond/Equity Fund (GBEF) portfolio. Specifically, here are the capital market lines:

(a) Lending and Borrowing against the Cat Bond/Equity Fund (CBEF) portfolio:

$$E(r_p) = r_f + \left[\frac{E(r_{CBEF}) - r_f}{\sigma_{CBEF}}\right]\sigma_p = \ .03 \ + \ (.07/.1188)\,\sigma_p.$$

(b) Lending and Borrowing against the Government Bond/Equity Fund (GBEF) portfolio:

$$E(r_p) = r_f + \left[\frac{E(r_{GBEF}) - r_f}{\sigma_{GBEF}}\right]\sigma_p = \ .03 \ + \ (.07/.1469)\,\sigma_p.$$

These equations imply that for all possible portfolio risk levels (σ_p), expected return will be higher if you lend/borrow against the Cat Bond/Equity Fund portfolio rather than the Government Bond/Equity Fund portfolio.

Problem 2.

Suppose that you are analyzing the risk and return characteristics of two securities, A and B. You have estimated that these two securities will provide the following set of state-contingent returns $r_{A,s}$ and $r_{B,s}$ (note: p_s represents the probability that state s will occur):

State of Economy	p_s	$r_{A,s}$	$r_{B,s}$
Boom	50.00%	5.00%	40.00%
Bust	50.00%	10.00%	-5.00%

A. What are the expected returns for securities A and B?

Solution:

$$E(r_A) = \sum_{s=1}^{n} p_s r_{As} = .5(.05) + .5(.10) = 7.5\%.$$

$$E(r_B) = \sum_{s=1}^{n} p_s r_{Bs} = .5(.40) + .5(-.05) = 17.5\%.$$

B. What are the standard deviations of the returns for securities A and B?

Solution:

$\sigma_A^2 = \sum_{s=1}^{n} p_s (r_{As} - E(r_A))^2 = .5(.05 - .075)^2 + .5(.10 - .075)^2 = .0625\%$; therefore, $\sigma_A =$ 2.50%;

$\sigma_B^2 = \sum_{s=1}^{n} p_s (r_{Bs} - E(r_B))^2 = .5(.40 - .175)^2 + .5(-.05 - .175)^2 = 5.0625\%$; therefore, $\sigma_B = 22.50\%$;

Note: Since there are equal probabilities of Boom and Bust, the standard deviation of each security can also be determined simply by calculating half of the total dispersion between the Boom and Bust states; thus, σ_A= 0.5(.10 - .05) = 2.5%, and σ_B= 0.5(.40 + .05) = 22.5%.

C. Suppose that state-contingent returns on the risk-free asset ($r_{f,s}$) and the market portfolio ($r_{m,s}$) are estimated as follows:

State of Economy	p_s	$r_{f,s}$	$r_{m,s}$
Boom	50.00%	5.00%	25.00%
Bust	50.00%	5.00%	0.00%

Combine the state-contingent return data for the risk-free asset and the market portfolio with the state-contingent return data presented earlier in this problem for securities A and B. Is security A underpriced, correctly priced, or overpriced? Also, determine whether security B is underpriced, correctly priced, or overpriced. Justify your answers by comparing expected returns calculated in part A of this problem with expected returns based on the Capital Asset Pricing Model (CAPM).

Solution: In order to determine whether Securities A and B are underpriced, overpriced, or correctly priced, we need to calculate the beta coefficients for each security. Since $\beta_i = \sigma_{im}/\sigma_m^2$, this requires calculating the covariances between A and the market and between B and the market, and then dividing each of these numbers by the variance of the market.

First, we find the variance of the market:

$$\sigma_m^2 = \sum_{i=1}^{n} p_s(r_{ms} - E\left(r_m\right))^2 = .5(.25 - .125)^2 + .5(0 - .125)^2 = \left[1/2\left(.25 - .0\right)\right]^2 = 1.5625\%.$$

Next, we find the covariances:

$$\begin{aligned}\sigma_{Am} &= \sum_{s=1}^{n} p_s(r_{As} - E(r_A))(r_{ms} - E(r_m)) \\ &= .5\left(.05 - .075\right)\left(.25 - .125\right) + .5\left(.10 - .075\right)\left(0 - .125\right) = -.0031\end{aligned}$$

$$\begin{aligned}\sigma_{Bm} &= \sum_{s=1}^{n} p_s(r_{Bs} - E(r_B))(r_{ms} - E(r_m)) \\ &= .5\left(.40 - .175\right)\left(.25 - .125\right) + .5\left(-.05 - .175\right)\left(0 - .125\right) = .0281\end{aligned}$$

Therefore, $\beta_A = \dfrac{\sigma_{Am}}{\sigma_m^2} = \dfrac{-.0031}{.015625} = -.2$, and $\beta_B = \dfrac{\sigma_{Bm}}{\sigma_m^2} = \dfrac{.0281}{.015625} = 1.80.$

Consequently,

$$E(r_A) = r_f + [E(r_m) - r_f]\beta_A = 5\% + [12.5\% - 5\%]\,(-.20) = 3.5\%, \text{and}$$
$$E(r_B) = r_f + [E(r_m) - r_f]\beta_B = 5\% + [12.5\% - 5\%]\,(1.80) = 18.5\%.$$

Since security A should only provide an expected return of 3.5% but is currently priced to provide an expected return of 7.5%, this means that this security is underpriced. Investors will realize this and consequently there will be excess demand for security A, which will cause its price to increase and its expected return to fall until it is in line with the CAPM. On the contrary, security B should provide an expected return of 18.5% but is currently priced to provide an expected return of only 17.5%, which means that this security is overpriced. Investors will realize this and consequently there will be excess supply for security B, which will cause its price to decrease and its expected return to rise until it is in line with the CAPM.