

BAYLOR UNIVERSITY  
HANKAMER SCHOOL OF BUSINESS  
DEPARTMENT OF FINANCE, INSURANCE & REAL ESTATE

Risk Management  
Dr. Garven  
Problem Set 8

Name: \_\_\_\_\_ SOLUTIONS \_\_\_\_\_

**Problem 1** (50 points)

Suppose the current value of a (non-dividend-paying) stock is \$10,000, and the annual continuously compounded riskless rate of interest is 4%. Based on the example provided on pp. 9-14 from the “[Derivatives Theory, Part 1](#)” lecture note, solve parts A and B below.

- A. (25 points) What is the “arbitrage-free” price for a forward contract on this stock which matures 1 year from today?

SOLUTION:  $F = Se^{rT} = \$10,000e^{.04} = \$10,408.11$

- B. (25 points) Suppose the forward price is \$10,400. Describe a profitable zero risk, zero net investment trading strategy involving the forward contract and its replicating portfolio. If you implement such a strategy, how much profit will you earn?

SOLUTION: Since the forward price of \$10,400 is below its “arbitrage-free” price by \$8.11, it is undervalued. In order to take advantage of this mispricing in a way that involves zero risk and zero net investment, one should buy a 1-year forward contract for \$10,400, sell the stock for \$10,000, and lend \$10,000 at the riskless rate. The following table succinctly summarizes this trading strategy:

Transaction	Payoff now	Payoff @ $T$
Buy Forward	\$0	$S_T - \$10,400$
Sell Stock	\$10,000	$- S_T$
Lend	(\$10,000)	$\$10,000e^{.04} = \$10,408.11$
<b>Arbitrage Profit</b>	<b>\$0</b>	<b>\$8.11</b>

**Problem 2** (50 points)

The price of a share of Zoom stock is currently \$250. It is known that at the end of 1 year, the Zoom share price will be either \$312.50 or \$200. The riskless interest rate is 2% per year.

- A. (10 points) Calculate the price of a 1-year European call option on Zoom stock with an exercise price of \$250 by applying the replicating portfolio approach.

SOLUTION: According to the replicating portfolio approach:

$$\begin{aligned}
 C_u &= \text{Max}(0, S_u - K) = 62.50 \\
 C_d &= \text{Max}(0, S_d - K) = 0 \\
 V_{RP} &= \Delta S + B \\
 \Delta S &= \frac{C_u - C_d}{S(u - d)} S = \frac{62.50}{250(.45)} 250 = .5556(250) = 138.89 \\
 B &= \frac{uC_d - dC_u}{e^{r\delta t}(u - d)} = \frac{1.25(0) - .8(62.50)}{e^{.02(1)}(.45)} = -108.91 \\
 \therefore C = V_{RP} &= \Delta S + B = \$138.89 - \$108.91 = \$29.98.
 \end{aligned}$$

- B. (10 points) Calculate the price of a 1-year European call option on Zoom stock with an exercise price of \$250 by applying the risk neutral valuation approach.

SOLUTION: The risk neutral probability of an up move is  $q = \frac{e^{r\delta t} - d}{u - d} = \frac{e^{.02} - .8}{1.25 - .8} = .4893$ . Since the stock is worth  $\$250(1.25) = \$312.50$  at the  $u$  node and  $\$250(.8) = \$200$  at the  $d$  node, this means that the call is only in the money at the  $u$  node; specifically, it is worth \$62.50 at that node. Therefore, the price of a one-year call option is

$$C = e^{-r\delta t}[qC_u + (1 - q)C_d] = e^{-.02} [.4893(62.50)] = \$29.89.$$

- C. (10 points) Calculate the price of a 1-year European put option on Zoom stock with an exercise price of \$250.

SOLUTION: According to the put-call parity equation,  $C + Ke^{-r\delta t} = P + S$ ; therefore,  $P = C + Ke^{-r\delta t} - S \Rightarrow P = 29.98 + 250e^{-.02(1)} - 250 = \$25.03$ .

- D. (20 points) Next, add another 1-year timestep; i.e., it is known that at the end of 2 years, the Zoom share price will be \$390.63, \$250, or \$160. Calculate the price of a 2-year European call option on Zoom stock with an exercise price of \$250. Also calculate the price of a 2-year European put option on Zoom stock with an exercise price of \$250.

SOLUTION: Note that the call option is only in the money at the  $uu$  node, where it is worth \$140.63. It is worthless at the  $ud$  and  $dd$  nodes. Applying risk neutral valuation, the price of a two-year call option is

$$\begin{aligned}
 C &= e^{-2r\delta t}[q^2C_{uu} + 2q(1 - q)C_{ud} + (1 - q)^2C_{dd}] \\
 &= e^{-.04} [.4893^2(140.63)] = \$32.35.
 \end{aligned}$$

Regarding the (otherwise identical) 2-year put option, put-call parity indicates that the put is worth \$22.55:

$$P = C + Ke^{-2r\delta t} - S = 32.35 + \$250e^{-.04} - \$250 = \$22.55.$$