Statistics Extra Credit Problem Solutions

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Suppose the return distributions for two risky assets are:

State	p_s	$r_{a,s}$	$r_{b,s}$
1	1/3	-3%	36%
2	1/3	9%	-12%
3	1/3	21%	12%

1. Calculate the expected returns for assets a and b.

$$E(r_a) = \sum_{s=1}^{n} p_s r_{a,s} = (1/3)(-3\%) + (1/3)(9\%)(1/3)(21\%) = 9\%$$
$$E(r_b) = \sum_{s=1}^{n} p_s r_{b,s} = (1/3)(36\%) + (1/3)(-12\%)(1/3)(12\%) = 12\%$$

2. Calculate the variances and standard deviations for assets a and b.

$$\sigma_a^2 = \sum_{s=1}^{n} p_s (r_{a,s} - E(r_a))^2$$
$$= (1/3)(-3\% - 9\%)^2 + (1/3)(9\% - 9\%)^2 + (1/3)(21\% - 9\%)^2 = .96\%$$
$$\sigma_a = \sqrt{.96\%} = 9.8\%$$
$$\sigma_b^2 = \sum_{s=1}^{n} p_s (r_{a,s} - E(r_a))^2$$
$$= (1/3)(36\% - 12\%)^2 + (1/3)(-12\% - 12\%)^2 + (1/3)(12\% - 12\%)^2 = 3.84\%$$
$$\sigma_b = \sqrt{3.84\%} = 19.6\%$$

3. Calculate the covariance and correlation between assets a and b.

$$\sigma_{ab} = \sum_{s=1}^{n} p_s (r_{a,s} - E(r_a))(r_{b,s} - E(r_b))$$
$$= (1/3)(-12\%)(24\%) + (1/3)(0\%)(-24\%) + (1/3)(12\%)(0) = -.96\%$$
$$\rho_{ab} = \frac{\sigma_{ab}}{\sigma_a \sigma_b} = \frac{.96\%}{(9.8\%)(19.6\%)} = -.50$$

4. Calculate the expected return and standard deviation for an equally weighted portfolio comprising asset a and b.

$$E(r_p) = \sum_{s=1}^{n} w_i E(r_i) = .5(9\%) + .5(12\%) = 10.5\%$$
$$\sigma_p = \sqrt{w_a^2 \sigma_a^2 + w_b^2 \sigma_b^2 + 2 w_a w_b \sigma_{ab}}$$
$$= \sqrt{.25(.96\%) + .25(3.84\%) + 2(.5)(.5)(-.96\%)} = 8.49\%$$