Formula Sheet For The Statistics Class Problem

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Suppose the return distributions for two risky assets are:

State	p_s	$r_{a,s}$	$r_{b,s}$
1	1/3	-3%	36%
2	1/3	9%	-12%
3	1/3	21%	12%

1. Calculate the expected returns for assets *a* and *b*.

   Expected return for asset *a*: $E(r_a) = \sum_{s=1}^{n} p_s r_{a,s}$

   Expected return for asset *b*: $E(r_b) = \sum_{s=1}^{n} p_s r_{b,s}$

2. Calculate the variances and standard deviations for assets *a* and *b*.

   Variance for asset *a*: $\sigma_a^2 = \sum_{s=1}^{n} p_s (r_{a,s} - E(r_a))^2$ and standard deviation for asset *a*: $\sigma_a = \sqrt{\sigma_a^2}$

   Variance for asset *b*: $\sigma_b^2 = \sum_{s=1}^{n} p_s (r_{b,s} - E(r_b))^2$ and standard deviation for asset *b*: $\sigma_b = \sqrt{\sigma_b^2}$

3. Calculate the covariance and correlation between assets *a* and *b*.

   covariance between assets *a* and *b*: $\sigma_{ab} = \sum_{s=1}^{n} p_s (r_{a,s} - E(r_a))(r_{b,s} - E(r_b))$ and correlation between assets *a* and *b*: $\rho_{ab} = \frac{\sigma_{ab}}{\sigma_a \sigma_b}$

4. Calculate the expected return and standard deviation for an equally weighted portfolio comprising assets *a* and *b*.

   Define w_a as the percent of the portfolio allocated to asset *a*, and w_b as the percent of the portfolio allocated to asset *b*, where $w_b = 1 - w_a$. In the case of an equally weighted portfolio, $w_a = w_b = .5$. Then the expected return for an equally weighted portfolio comprising assets *a* and *b* is $E(r_p) = w_a E(r_a) + w_b E(r_b) = .5E(r_a) + .5E(r_b)$, and the standard deviation for such a portfolio is

   $\sigma_p = \sqrt{w_a^2 \sigma_a^2 + w_b^2 \sigma_b^2 + 2 w_a w_b \sigma_{ab}} = \sqrt{.25\sigma_a^2 + .25\sigma_b^2 + 2(.5)(.5)\sigma_{ab}}$

5. Determine the least risky combination of assets *a* and *b* and calculate the expected return and

standard deviation for such a portfolio.

Not that portfolio variance for a 2-asset portfolio is $\sigma_p^2 = w_a^2\sigma_a^2 + w_b^2\sigma_b^2 + 2w_a w_b \sigma_{ab}$. Since $w_b = 1 - w_a$, we replace all incidences of w_b in this equation with $1 - w_a$. Thus, $\sigma_p^2 = w_a^2\sigma_a^2 + (1-w_a)^2\sigma_b^2 + 2w_a(1-w_a)\sigma_{ab}$. We minimize σ_p^2 by differentiating σ_p^2 with respect to w_a, setting the resulting equation equal to 0, and solving for w_a. The asset allocation weighting scheme which minimizes σ_p^2 requires investing the proportion $w_a = \dfrac{\sigma_b^2 - \sigma_{ab}}{\sigma_a^2 + \sigma_b^2 - 2\sigma_{ab}}$ in asset *a*, and the proportion 1-*a* in asset *b*. All that differs between problems 4 and 5 is a different weighting scheme; therefore, we use the same equations for calculating portfolio expected return and standard deviation in problem 5 as we did in problem 4, and we find that expected return and risk of the equally weighted portfolio in problem 4 are both greater than the expected return and risk of the minimum variance portfolio in problem 5.