Option Pricing Class Problem Solutions (Parts A and B)

Finance 4335, March 28, 2019

Ripple, Inc., stock is currently worth \$56. Each year, it can change by a factor of 0.9 or 1.3. The stock pays no dividends, and the annual continuously compounded risk-free interest rate is 4%.

A. Calculate the price of a one-year European put option on Ripple, Inc. stock with an exercise price of \$60.

SOLUTION: We can solve this problem by via replicating portfolio, delta hedging and risk neutral valuation approaches.

1. According to the Replicating Portfolio Approach:

$\Delta = \frac{P_u - P_d}{uS - dS} = \frac{0 - 9.60}{72.80 - 50.40} = -.4286$; and $B = \frac{uP_d - dP_u}{e^{r\delta t}(u-d)} = \frac{1.3(9.60) - .9(0)}{1.0408(.4)} =$ 29.98. Then $V_{RP} = P = \Delta S + B = -.4286(56) + 29.98 = \$5.98$.

2. According to the Delta Hedging Approach:

$$\begin{aligned}
P_u &= Max(0, K - S_u) = 0 \\
P_d &= Max(0, K - S_d) = 9.6 \\
V_H &= P + \Delta S = P + \Delta 56. \\
V_H^u &= V_H^d \Rightarrow 0 + \Delta 72.80 = 9.60 + \Delta 50.40 \Rightarrow \Delta = .4286. \\
V_H^u &= V_H^d = 31.20 \\
V_H &= P + \Delta 56 = P + 24 = e^{-.04} 31.20 = .9608(31.20) = 29.98 \\
P &= \$5.98
\end{aligned}$$

3. According to the Risk Neutral Valuation Approach:

The risk neutral probability of an up move is $q = \frac{e^{r\delta t} - d}{u - d} = \frac{e^{.04} - .9}{1.3 - .9} = .352$. Since the stock is worth $\$56(1.3) = \$72.80$ at the u node and $\$56(.9) = \$50.40$ at the d node, this means that the put is only in the money at the d node; specifically, it is worth \$9.60 at that node. Therefore, the price of a one-year put option is

$$p = e^{-r\delta t}[qp_u + (1-q)p_d] = e^{-.04}[.648(9.60)] = \$5.98.$$

B. Calculate the price of a one-year European call option on Ripple, Inc. stock with an exercise price of \$60.

SOLUTION: We can solve this problem by via put-call parity, delta hedging, replicating portfolio, and risk neutral valuation approaches.

1. According to put-call parity,

$$C = P + S - Ke^{-r\delta t} = \$5.98 + \$56 - \$60e^{-.04} = \$4.33.$$

2. According to the Replicating Portfolio Approach:

$\Delta = \dfrac{C_u - C_d}{uS - dS} = \dfrac{12.80 - 0}{72.80 - 50.40} = .5714$ and $B = \dfrac{uC_d - dC_u}{e^{r\delta t}(u-d)} = \dfrac{1.3(0) - .9(12.80)}{1.0408(.4)} = -27.67$. Then $V_{RP} = C = \Delta S + B = .5714(56) - 27.67 = \$4.33.$

3. According to the Delta Hedging Approach:

$$
\begin{aligned}
C_u &= Max(0, S_u - K) = 12.80 \\
C_d &= Max(0, S_d - K) = 0 \\
V_H &= C - \Delta S = C - \Delta 56. \\
V_H^u &= V_H^d \Rightarrow 12.80 - \Delta 72.80 = 0 - \Delta 50.40 \Rightarrow \Delta = .5714. \\
V_H^u &= V_H^d = -28.80 \\
V_H &= C - \Delta 56 = C - 32 = -e^{-.04} 28.80 = -.9608(28.80) = -27.67 \\
C &= \$4.33
\end{aligned}
$$

4. According to the Risk Neutral Valuation Approach:

The risk neutral probability of an up move is $q = \dfrac{e^{r\delta t} - d}{u - d} = \dfrac{e^{.04} - .9}{1.3 - .9} = .352$. Since the stock is worth $\$56(1.3) = \$72.80$ at the u node and $\$56(.9) = \$50.40$ at the d node, this means that the call is only in the money at the u node; specifically, it is worth \$12.80 at that node. Therefore, the price of a one-year call option is

$$c = e^{-r\delta t}[qc_u + (1-q)c_d] = e^{-.04}[.352(12.80)] = \$4.33.$$