Midterm Exam #2 Formula Sheet

Portfolio Theory Concepts

σ_i = standard deviation of returns on asset i;
σ_{ij} = covariance between i and j;
ρ_{ij} = correlation between i and j = $\sigma_{ij}/\sigma_i\sigma_j$;
w_i = proportion of portfolio p invested in asset i (note: $\sum_{i=1}^{n} w_i = 1$);
$E(r_p)$ = expected portfolio return $= \sum_{i=1}^{n} w_i E(r_i)$; and
σ_p^2 = portfolio variance $= \sum_{i=1}^{n}\sum_{j=1}^{n} w_i w_j \sigma_{ij}$; when n=2, $\sigma_p^2 = w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1w_2\sigma_{ij}$.
When n=2, $w_1 = \dfrac{\sigma_2^2 - \sigma_{12}}{\sigma_1^2 + \sigma_2^2 - 2\sigma_{12}}$ and $w_2 = 1 - w_1$.

Put-call parity theorem (assuming no dividends; options are European):

$$c + Ke^{-rn\delta t} = p + S,$$

where
δt = the length of a timestep (e.g., if the timestep is one year, then $\delta t = 1$);
n = the number of timesteps until expiration;
c = value of a call option with an exercise price of K and n timesteps until expiration;
p = value of a put option with an exercise price of K and n timesteps until expiration;
r = the annualized riskless rate of interest; and
S = the value of the underlying asset.

Risk neutral probability of an "up" move:

$$q = \frac{e^{r\delta t} - d}{u - d},$$

where
q = the risk neutral probability of an "up" move;
u = 1 plus the rate of return from an "up" move; and
d = 1 plus the rate of return from a "down" move.

Risk Neutral Valuation Formula (for a 1 timestep call or put option):

$$f = e^{-r\delta t}[qf_u + (1-q)f_d],$$

where f_u = value of option at node u and f_d = value of option at node d .

Risk Neutral Valuation Formula for an n timestep European call option (AKA the "Cox-Ross-Rubinstein" model):

$$c = e^{-rn\delta t}\left[\sum_{j=a}^{n}\left(\frac{n!}{j!\,(n-j)!}\right)q^j(1-q)^{n-j}\left(u^j d^{n-j}S-K\right)\right],$$

where $a =$ the smallest integer value $> \ln(K/Sd^n)/\ln(u/d)$.

Replicating Portfolio Parameters for call and put options:

$$\text{At the tree's inception, } \Delta = \frac{f_u - f_d}{uS - dS} \text{ and } B = \frac{uf_d - df_u}{e^{r\delta t}(u-d)},$$
$$\text{At node } u,\ \Delta_u = \frac{f_{uu} - f_{ud}}{u^2S - udS} \text{ and } B_u = \frac{uf_{ud} - df_{uu}}{e^{r\delta t}(u-d)}, \text{ and}$$
$$\text{At node } d,\ \Delta_d = \frac{f_{ud} - f_{dd}}{udS - d^2S} \text{ and } B_d = \frac{uf_{dd} - df_{ud}}{e^{r\delta t}(u-d)}.$$

In the equations above, Δ, Δ_u, and Δ_d correspond to the number of units of the underlying asset which are either bought or sold at each node on the binomial tree, whereas B, B_u, and B_d correspond to the values of the corresponding short or long bond positions at each node. Thus, the value of the replicating portfolio at the tree's inception is $V_{RP} = \Delta S + B = f$, and at nodes $u$and d, $V_{RP}^u = \Delta_u S_u + B_u = f_u$ and $V_{RP}^d = \Delta_d S_d + B_d = f_d$ respectively.