Baylor University
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Risk Management
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Midterm Exam 2
Fall 2018

Name: SOLUTIONS

Notes:

1. This test consists of 4 problems. Select three out of four from Problems 1-4. At your option, you may complete all four problems, in which case I will count the three highest scoring problems. Since the maximum number of points possible for three problems is 96, I will award everyone who takes this exam an extra 4 points; thus the maximum number of points on this exam is 100. :-)

2. You may have the entire class period in order to complete this examination. Be sure to show your work as well as provide a complete answer for each problem; i.e., in addition to producing numerical results, also explain your results in plain English.

Good luck!

Problem 1 (32 points)

Consider an economy with three kinds of drivers: safe, dangerous, and crazy. There is an equal number of each of these drivers, and each driver has initial wealth of \$150 and utility $U(W) = \sqrt{W}$. In any given year, a safe driver gets into an accident with probability $p_s = 0.1$; a dangerous driver gets into an accident with probability $p_d = 0.2$, and a crazy driver gets into an accident with probability $p_c = 0.3$. An accident leads to repair costs of \$50, and has no other consequences.

Suppose that an insurance company offers full coverage insurance policies to these drivers; i.e., when accidents occur, claim payments of \$50 are made which fully cover repair costs. The drivers decide whether to purchase these full coverage policies.

A. (8 points) Suppose the insurance company can distinguish the three types of drivers, and offers each driver an actuarially fair full coverage insurance policy. Which of the drivers will buy such a policy?

SOLUTION: If insurance is actuarially fair, we know from the Bernoulli hypothesis that risk averse policyholders will prefer full coverage.

Simply invoking the Bernoulli hypothesis is sufficient for earning full credit. However, it is also acceptable if the student demonstrates that the safe, dangerous, and crazy drivers all have higher expected utility when insurance is actuarially fair. Actuarially fair insurance for the safe drivers costs $E(L_s) = \sum_{s=1}^{n} p_{s,s} L_s = .1(50) = \$5$. For dangerous drivers, actuarially fair insurance costs $E(L_d) = \sum_{s=1}^{n} p_{s,d} L_s = .2(50) = \$10$, and for crazy drivers, actuarially fair insurance costs $E(L_c) = \sum_{s=1}^{n} p_{s,c} L_s = .3(50) = \$15$.

Next, we compute expected utility when there is no insurance:

- EU (uninsured safe driver): $E(U(W_s)) = \sum_{s=1}^{n} p_{s,s} U(W_s) = .1\sqrt{100} + .9\sqrt{150} = 12.0227$.
- EU (uninsured dangerous driver): $E(U(W_d)) = \sum_{s=1}^{n} p_{s,d} U(W_s) = .2\sqrt{100} + .8\sqrt{150} = 11.7980$.
- EU (uninsured crazy driver): $E(U(W_c)) = \sum_{s=1}^{n} p_{s,c} U(W_s) = .3\sqrt{100} + .7\sqrt{150} = 11.5732$.

Expected utility is always higher when these drivers can fully insure at actuarially fair prices:

- EU (insured safe driver): $E(U(W_s)) = \sqrt{145} = 12.0416$.
- EU (insured dangerous driver): $E(U(W_d)) = \sqrt{140} = 11.8322$.
- EU (insured crazy driver): $E(U(W_c)) = \sqrt{135} = 11.6190$.

B. (8 points) Now suppose that the insurance company cannot distinguish the three types of drivers. Therefore, it decides to offer to offer a full coverage insurance policy to all drivers for a price of $10. Show that safe drivers would not be willing to purchase such a policy, whereas dangerous and crazy drivers would purchase such a policy.

SOLUTION: If a full coverage policy costs $10, this means that the expected utility of full coverage is $\sqrt{140}$= 11.8322 for everyone. Since safe drivers have higher expected utility when they remain uninsured, they will not be willing to purchase this policy. We have already determined that the dangerous drivers will insure at a price of $10, so we know that they will stay in the market. The crazy drivers are certainly quite happy about this deal; they would have insured at a price of $15 but are now getting an even better deal. However, now the insurer will lose money because the crazy drivers have an expected cost of $15 and the insurer is only collecting $10 in premium from each of these drivers.

C. (8 points) Since safe drivers are not willing to purchase the policy described in part B, the insurance company decides to increase the price of the policy so that it won't lose money. This time, it offers a full coverage insurance policy for a price of $12.50. Show that safe and dangerous drivers would not be willing to purchase such a policy, whereas crazy drivers would purchase such a policy.

SOLUTION: If a full coverage policy costs $12.50, this means that the expected utility of full coverage is $\sqrt{137.50}$= 11.7260 for the dangerous and crazy drivers. Since dangerous drivers have higher expected utility when they remain uninsured, they will not be willing to purchase this policy. We have already determined that the dangerous drivers will insure at a price of $10, so we know that they will stay in the market. However, now the insurer will lose money because the crazy drivers have an expected cost of $15 and the insurer is only collecting $12.50 in premium from each of these drivers.

D. (8 points) This problem numerically illustrates how markets can fail due to adverse selection; i.e., even though safe and dangerous drivers are risk averse and would like to purchase fairly priced insurance, the market will only provide fairly priced insurance for the crazy drivers. Explain the intuition for why this occurs.

SOLUTION: The basic dilemma here is that when the insurer tries to charge an average premium, then the low risk insureds drop out of the risk pool, which in turn raises the average cost of providing insurance for the remaining policyholders. The insurer gets stuck in a vicious cycle; when she raises premiums in response to the lower risk policyholders dropping out, this in turn aggravates the problem even more. This process continues until only the worst risks are left.

Problem #2 (32 points)

Suppose that you are interested in investing in a portfolio consisting of stocks and bonds. You have already decided to invest in an equity fund that is indexed to the S&P 500, but are not sure whether to invest in Government bonds or Catastrophe ("Cat") bonds. The Cat bonds are riskier than Government bonds because Cat bond investors must forfeit interest and principal payments in the event that a Category 4 or 5 hurricane hits Florida. The following table summarizes the expected returns and standard deviations for these securities:

Security	**Expected Return**	**Standard Deviation**
Cat Bonds	8%	15%
Equity Fund	12%	20%
Gov't Bonds	5%	8%

The correlation between Cat Bond returns and Equity Fund returns is -.1, whereas the correlation between Government Bond returns and Equity Fund returns is .10.

A. (10 points) Suppose you would like to earn 10% on your portfolio. If your portfolio consisted of Cat Bonds and the Equity Fund, what would be its (the portfolio's) standard deviation? (Hint: calculate the security weights that would generate a 10% expected return on a portfolio that comprises the Cat Bonds and the Equity Fund, and then use these weights to calculate standard deviation).

SOLUTION: The expected return on a portfolio consisting of two assets is $E(r_p) = w_1E(r_1) + w_2E(r_2) = w_1E(r_1) + w_2(1 - E(r_2))$. The expected return on the Cat Bond, $E(r_1) = 8\%$, and the expected return on the Equity Fund, $E(r_2) = 12\%$. Therefore, $E(r_p) = w_1(.08) + (1 - w_1)(.12) = 10\% \Rightarrow w_1 = w_2 = 50\%$, and

$$\sigma_p = \sqrt{w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1w_2\sigma_{12}}$$
$$= \sqrt{.50^2\left(.15^2\right) + .50^2\left(.20^2\right) + 2\,(.5)\,(.5)\,(-.1)\,(.15)\,(.2)} = 11.88\%.$$

B. (10 points) What is the standard deviation of a portfolio consisting of Government Bonds and the Equity Fund that would have an expected return of 10%?

SOLUTION: The expected return on the Government Bond, $E(r_1) = 5\%$, and the expected return on the Equity Fund, $E(r_2) = 12\%$. Therefore, $E(r_p) = w_1(.05) + (1 - w_1)(.12) = 10\% \Rightarrow w_1 = 2/7 \Rightarrow w_2 = 5/7$, and

$$\sigma_p = \sqrt{w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1w_2\sigma_{12}}$$
$$= \sqrt{(2/7)^2\left(.08^2\right) + (5/7)^2\left(.20^2\right) + 2\,(2/7)\,(5/7)\,(.10)\,(.08)\,(.2)} = 14.69\%.$$

C. (12 points) Suppose the riskless rate of interest is 3%, and you can borrow or lend against either of the portfolio combinations described in parts A and B above. Which of these two portfolio combinations is optimal for arbitrarily risk averse investors? Justify your answer.

SOLUTION: The optimal portfolio combination is the Cat Bond/Equity Fund portfolio described in part A of this problem. The reason why this is optimal is that the slope of the capital market line that is generated from lending and borrowing against this portfolio is steeper than the slope of the capital market line that is generated from lending and borrowing against the Government Bond/Equity Fund portfolio. Specifically, here are the capital market lines:

- Lending and Borrowing against the Cat Bond/Equity Fund portfolio:

$$E(r_p) = r_f + \left[\frac{E(r_m) - r_f}{\sigma_m}\right] \sigma_p = .03 + \ (.07/.1188)\, \sigma_p.$$

- Lending and Borrowing against the Government Bond/Equity Fund portfolio:

$$E(r_p) = r_f + \left[\frac{E(r_m) - r_f}{\sigma_m}\right] \sigma_p = .03 + \ (.07/.1469)\, \sigma_p.$$

These equations imply that for all possible portfolio risk levels (σ_p), expected return will be higher if you lend/borrow against the Cat Bond/Equity Fund portfolio rather than the Government Bond/Equity Fund portfolio.

Problem 3 (32 points)

Consider a European call option on a non-dividend-paying stock where the stock price is \$90, the exercise price is \$100, the (annualized) risk-free rate is $r = 3\%$, $u = 1.1$, $d = 1/u = 1/1.1$, the number of timesteps is 4 (hint: each timestep occurs over a three month time interval), and the time to expiration is one year.

A. (16 points) What is the arbitrage-free price for this call option?

SOLUTION: The easiest method for finding the price of this call option is to apply the Cox-Ross-Rubinstein binomial option pricing model. The first step involves determining the minimum number of up moves (denoted as "a") required in order for the call option to expire in-the-money; since a is the smallest integer value $> \ln(K/Sd^n)/\ln(u/d) = \ln(100/(90(1/1.1)^4))/\ln(1.1^2) = 2.55$, this implies that $a = 3$; thus, the call option will be in the money after 3 and 4 up moves (i.e., at nodes $uuud$ and $uuuu$). Thus,

$$c_{uuuu} = \max[0, u^4S - K] = 1.1^4(90) - 100 = \$31.77; \text{ and}$$
$$c_{uuud} = \max[0, u^3dS - K] = 1.1^3(.9091)(90) - 100 = \$8.90.$$

The risk neutral probability of one up move is

$$q = \frac{e^{r\delta t} - d}{u - d} = \frac{e^{.03(.25)} - .9091}{1.1 - .9091} = .5156.$$

Applying risk neutral valuation, we find that

$$c = e^{-4r\delta t}[q^4c_{uuuu} + 4q^3(1-q)c_{uuud}] = e^{-.03}[.5156^4(31.77) + 4(.5156^3)(.4844)(8.90)] = \$4.47.$$

B. (16 points) What is the current market value of an otherwise identical (i.e., same underlying asset, same strike price, interest rate, same volatility, same number of timesteps, and time to expiration) European put option?

SOLUTION: Applying the put-call parity equation, $p = c + Ke^{-rn\delta t} - S = \$4.47 + \$100e^{-.03} - \$100 = \$11.52$.

Problem 4 (32 points)

Assume that the ABC stock price over the next two months is described by a two-period binomial model with u =1.05 and d =0.95. Each period represents one month. The riskless rate of interest is 5% per year. Assume that ABC stock is trading at \$100 per share now and that ABC will not pay any dividends during the course of the next two months.

A. (10 points) What is today's price of a European call option on ABC stock that expires in two months and has an exercise price of $K = \$100$?

SOLUTION: There are a number of ways to solve this problem; e.g., the delta hedging, replicating portfolio, or risk neutral valuation approaches can be applied in conjunction with backward induction. However, since the option is European and may only be exercised after two time-steps, the simplest method is to determine nodes after two time-steps in which the call is in the money and then apply risk neutral valuation to pricing the option payoffs at these particular nodes (this is the so-called "Cox-Ross-Rubinstein" approach). So, we begin by writing out the binomial tree for the underlying stock:

		\$110.25
	\$105.00	
\$100.00		\$99.75
	\$95.00	
		\$90.25

By inspection, the call option is only in the money at nodes uu. Since the risk neutral probability of an up move is $q = \frac{e^{r\delta t} - d}{u - d} = \frac{e^{.05/12} - .95}{1.05 - .95} = .5418$ and the risk neutral probability of arriving at node uu is q^2, today's "arbitrage-free" price for this call option is:

$$c = e^{-r2\delta t}[q^2 c_{uu}] = e^{-.05/6}[(.5418^2)(10.25) = \$2.98.$$

B. (10 points) Apply either the put-call parity equation or the risk neutral valuation equation to determine today's price of a European put option on ABC stock that expires in two months and has an exercise price of $K = \$100$.

SOLUTION: Since we know the prices for the call, the riskless bond, and the share, we can infer the arbitrage-free put option price from the put-call parity equation:

$$c + Ke^{-r2\delta t} = p + S \Rightarrow p = c + Ke^{-r2\delta t} - S = 2.98 + 100e^{-.05/6} - 100 = \$2.15.$$

We can also determine the arbitrage-free put option price via risk neutral valuation. By inspection, the put option is in the money at nodes ud and dd. Since the risk neutral probabilities of arriving at the ud and dd nodes are $2q(1-q)$ and $(1-q)^2$ respectively, today's "arbitrage-free" price for this put option is:

$$p = e^{-r2\delta t}[2q(1-q)^2 p_{ud} + (1-q)^2 p_{dd}] = e^{-.05/6}[2(.5418)(.4582)(.25) + .4582^2(9.75)] = \$2.15.$$

C. (12 points) Apply the replicating portfolio approach to determine today's price of a European put option on ABC stock that expires in two months and has an exercise price of $K = \$100$. (Hint: this requires calculating the arbitrage-free values of the replicating portfolio at nodes u, d, and today.)

SOLUTION:

- At node u, $\Delta_u = \dfrac{p_{uu} - p_{ud}}{u^2S - udS} = \dfrac{0 - .25}{110.25 - 99.75} = -.0238$ and $B_u = \dfrac{up_{ud} - dp_{uu}}{e^{r\delta t}(u-d)} = \dfrac{1.05(.25) - 0}{e^{.05/12}(1.05 - .95)} = \$2.61$. Since $S_u = \$105$, the value of the replicating portfolio at the up node is $V^u_{RP} = \Delta_u S_u + B_u = -.0238(105) + 2.61 = \$0.11$.
- At node d, $\Delta_d = \dfrac{p_{ud} - p_{dd}}{udS - d^2S} = \dfrac{.25 - 9.75}{99.75 - 90.25} = -1$ and $B_d = \dfrac{up_{dd} - dp_{ud}}{e^{r\delta t}(u-d)} = \dfrac{1.05(9.75) - .95(.25)}{e^{.05/12}(1.05 - .95)} = \$99.58$. Since $S_d = \$95$, the value of the replicating portfolio at the up node is $V^d_{RP} = \Delta_d S_d + B_d = -1(95) + 99.58 = \$4.58$.
- Today, $\Delta = \dfrac{p_u - p_d}{uS - dS} = \dfrac{.11 - 4.58}{105 - 95} = -.447$ and $B = \dfrac{up_d - dp_u}{e^{r\delta t}(u-d)} = \dfrac{1.05(4.58) - .95(.11)}{e^{.05/12}(1.05 - .95)} = \$46.85$. Since $S = \$100$, the value of the replicating portfolio today is $V_{RP} = \Delta S + B = -.447(100) + 46.85 = \$2.15$.