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Risk Management
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Problem Set 7

Name: SOLUTIONS

Problem 1

The following table lists the state-contingent returns on Security A ($r_{A,s}$) and Security B ($r_{B,s}$):

State of Economy	p_s	$r_{A,s}$	$r_{B,s}$
Bust	50%	-0.2000	+0.2500
Boom	50%	+0.4000	-0.0500

A. What are the expected returns for Security A and Security B?

SOLUTION: $E(r_A) = \sum_{s=1}^{n} p_s r_{A,s} = .5(-.2) + .5(.4) = .10$; and

$E(r_B) = \sum_{s=1}^{n} p_s r_{B,s} = .5(.25) + .5(-.05) = .10$.

B. What are the standard deviations of the returns for Security A and Security B?

SOLUTION:

$\sigma^2_{r_A} = \sum_{s=1}^{n} p_s (r_{A,s} - E(r_A))^2 = .5(-.2 - .1)^2 + .5(.4 - .1)^2 = .09$; therefore $\sigma_{r_A} = .3$; and

$\sigma^2_{r_B} = \sum_{s=1}^{n} p_s (r_{B,s} - E(r_B))^2 = .5(.25 - .1)^2 + .5(-.05 - .1)^2 = .0225$; therefore $\sigma_{r_B} = .15$.

C. Find the expected return and standard deviation for least possible risky combination of Security A and Security B. What is the composition of this portfolio (i.e., find the security weights w_A and w_B)?

SOLUTION:

$\sigma_{AB} = \sum_{s=1}^{n} (r_{As} - E(r_A))(r_{Bs} - E(r_B)) = .5(-.2-.1)(.25-.1) + .5(.4-.1)(-.05-.1) = -.045$.

Therefore, $w_A = \frac{\sigma^2_B - \sigma_{AB}}{\sigma^2_A + \sigma^2_B - 2\sigma_{AB}} = \frac{.0225 + .045}{.09 + .0225 + .09} = 1/3$. Consequently, $w_B = 1 - w_A = 2/3$, and $r_{mvp} = .10$. Furthermore, since $\rho_{AB} = -.045/(.3)(.15) = -1$, the standard deviation of the least possible risky combination of security A and security B is zero.

D. Suppose your initial wealth is \$100 and that you can borrow or lend up to \$100 at the riskless rate of interest of 5% during the course of the next year. Given this information, describe the most profitable *riskless* trading strategy which can be implemented, and calculate the profit from implementing this strategy.

SOLUTION: The most obvious riskless investment strategy would involve investing your initial wealth of \$100 in a riskless bond which yields 5%. Furthermore, since you can borrow up to \$100, you could also lever this strategy by investing \$200 at 5% and then paying back the principal plus interest on the \$100 loan. However, since the opportunity cost of capital for a riskless investment *is* the riskless rate of interest, these strategies do not increase your net wealth; i.e., their net present values are \$0.

It is possible to increase your net wealth without taking any risk by investing your initial wealth of \$100 plus an additional \$100 of borrowed money in the minimum variance portfolio; the value of such an investment after 1 year is $\$200e^{.1} - 100e^{.05}$= \$221.03 – \$105.13 = \$115.90, which implies an expected return totaling 15.9%. The net present value of this riskless arbitrage strategy is $NPV = \$115.90e^{-.05}$ - \$100 = \$10.25.

Problem 2

Suppose you have two stocks in your portfolio, *Max* and *Min*. The expected return of *Max* is 20% and the expected return of *Min* is 10%. The standard deviation of *Max* is 40% and the standard deviation of *Min* is 30%. The correlation between the two securities is zero. Suppose the riskless asset has an expected return of 4%.

A. What is the mean and standard deviation of the minimum variance portfolio combination of *Max* and *Min*?

SOLUTION: The ratio given by $w_{\text{Min}} = \dfrac{\sigma^2_{Max} - \sigma_{Max,Min}}{\sigma^2_{Min} + \sigma^2_{Max} - 2\sigma_{Max,Min}}$ provides a value for w_{Min} which minimizes portfolio variance; therefore, $w_{\text{Min}} = \dfrac{1,600}{900 + 1,600} = .64$; therefore, $E(r_p) = .36 \times E(r_{Max}) + .64 \times E(r_{Min}) = .36\,(20\%)\; + .64\,(10\%)\; = 13.6\%$, and $\sigma_p = \sqrt{.36^2 \times \sigma^2_{Max} + .64^2 \times \sigma^2_{Min} + 2 \times (.36)(.64)\sigma_{Max,Min}} =$ $\sqrt{.36^2 \times 1,600 + .64^2 \times 900} = 24\%$.

B. Which has the highest Sharpe ratio, *Max*, *Min* or the minimum variance portfolio combination of *Max* and *Min*?

SOLUTION: The Sharpe ratio is computed as the excess return on the security divided by its standard deviation. Therefore,

Sharpe Ratio$_{Max}$ = (20-4)/40 = 40%;

Sharpe Ratio$_{Min}$ = (10-4)/30 = 20%; and

Sharpe Ratio$_{.64/.36}$ = (13.6-4)/24 = 40%.

Therefore, *Max* and the minimum variance portfolio combination of *Max* and *Min* have the same Sharpe ratios (40%). The Sharpe ratio for *Min* is significantly lower (only 20%).

C. Suppose the correlation between *Max* and *Min* is -1. If this were the case, there would be an arbitrage opportunity, since a combination of *Max* and *Min* exists that is riskless and yields a higher expected return than the riskless asset. Describe the characteristics of a portfolio strategy that would enable you to generate positive profits without having to bear any risk or put up any of your own money. Assume that there are no restrictions on short sales or margin requirements.

SOLUTION: $w_{Min} = \dfrac{1,600 - (-1)(30)(40)}{900 + 1,600 - (-2)(30)(40)}$ =2,800/4,900 = 4/7. The expected return for this portfolio is $E(r_p) = \frac{3}{7} \times E(r_{Max}) + \frac{4}{7} \times E(r_{Min}) = .5714(10\%) + .4286(20\%) = 14.29\%$, and $\sigma_p = 0$ because $\rho_{Min,Max} = -1$. We can generate positive profits without having to bear any risk or put up any money by simply choosing the following set of weights: $w_{Min} = 4/7$, $w_{Max} = 3/7$, and $w_{r_f} = -1$. In other words, we go long 100 percent in the riskless combination of *Max* and *Min*, and 100 percent short in the riskless asset; i.e., we fund our investment in the combination of *Max* and *Min* by borrowing an equivalent sum of money at the riskless rate of interest.

D. Now suppose the expected return to the market portfolio is 12%, and the standard deviation of the market portfolio is 20%. Assuming that the CAPM holds, what are the betas for *Max* and *Min*?

SOLUTION: According to the CAPM, $E(r_{Max}) = r_f + [E(r_m) - r_f]\,\beta_{Max}$; therefore, $\beta_{Max} = \left(\dfrac{[E(r_{Max}) - r_f]}{[E(r_m) - r_f]}\right)$ = (20-4)/8 = 2. Similarly we can find β_{Min} by calculating the ratio $\left(\dfrac{[E(r_{Min}) - r_f]}{[E(r_m) - r_f]}\right)$ =(10-4)/8 = .75.