Pricing Credit Risk using the BSM model

Finance 4335 Class Problem, April 15, 2021

Problem Setup:

- Suppose two banks exist which are identical in all respects except for degree of financial leverage.
- At date $t = 0$, Bank 1 issues zero coupon deposits with a face value of \$500,000, whereas bank 2 has issued zero coupon deposits with a face value of \$800,000.
- Current ($t = 0$) asset value for both banks is \$1,000,000, and 1 year from today (at date $t = 1$), depositors expect these banks to pay back the face value of deposits with profits earned from their investments.
- However, since both banks are limited liability corporations, and hold risky assets ($\sigma = .4$), depositors face the risk of default.
- The annual riskless rate of interest is 3%.

1. Suppose there is no deposit insurance. What are the fair market values for the deposits held by Bank 1 and Bank 2 if there is no deposit insurance?

   Solution: In the absence of deposit insurance, depositors are at risk if default occurs; i.e., if $F < B$ at t =1. Consequently, the fair market value of risky deposits is equal to the fair market value of safe deposits minus the value of the limited liability put option; i.e., $V(D) = Be^{-rT} - V(Max[0, B - F])$, where B corresponds to the promised payment and F corresponds to the $t = 1$ value of bank assets.

   We begin by calculating the fair market value for Bank 1 deposits. The value of the limited liability put is $V(Max[0, B - F]) = Be^{-rT}N(-d_2) - V(F)N(-d_1)$,

$d_1 = \frac{\ln(V(F)/B) + (r + .5\sigma^2)T}{\sigma\sqrt{T}} = \frac{\ln(1,000,000/500,000) + (.03 + .5(.4^2))1}{.4\sqrt{1}} =$ 2.0079, and $d_2 = d_1 - \sigma\sqrt{T} = 2.0079 - .4\sqrt{1} = 1.6079$. Since $N(-d_1) = N(-2.0079) = .0223$ and $N(-d_2) = N(-1.6079) = .0539$, the value of Bank 1's limited liability put option is $V(Max[0, B-F]) = 500,000e^{-.03}(.0539) - 1,000,000(.0223) = \$3,840.40$, and the fair market value of Bank 1's deposits is $V(D) = Be^{-rT} - V(Max[0, B-F]) = 500,000e^{-.03} - \$3,840.40 = \$481,382.37$.

Next, we perform the same calculations for Bank 2. Bank 2's $d_1 = \frac{\ln(1,000,000/800,000) + (.03 + .5(4^2))1}{.4\sqrt{1}} = .8329$ and $d_2 = d_1 - \sigma\sqrt{T} = .8329 - .4\sqrt{1} = .4329. d_2 = d_1 - \sigma\sqrt{T} = 2.0079 - .4\sqrt{1} = 1.6079$. Since $N(-d_1) = N(-.08329) = .2025$ and $N(-d_2) = N(-.4329) = .3326$, the value of Bank 2's limited liability put option is $V(Max[0, B - F])$ $= 800,000e^{-.03}(.3326) - 1,000,000(.2025) = \$55,721.88$, and the fair market value of Bank 2's deposits is $V(D) = Be^{-rT} - V(Max[0, B - F]) = 800,000e^{-.03} - \$55,721.88 = \$720,634.55$.

2. What are the values of Bank 1 and Bank 2 limited liability put options?

   Solution: As calculated in the answer to question (1), the values of the limited liability put options held by Bank 1 and Bank 2 are \$3,840.40 and \$55,721.88 respectively.

3. What are the (risk neutral) probabilities of default for Bank 1 and Bank 2?

   Solution: As calculated in the answer to question (1), the (risk neutral) probabilities of default are $N(-d_2) = N(-1.6079) = .0539$ for Bank 1 and $N(-d_2) = N(-.4329) = .3326$ for Bank 2.

4. Calculate yields to maturity and credit risk premiums for Bank 1 and Bank 2.

   Solution: Since $B = V(D)e^{YTM(T)}$, it follows that $YTM = \frac{\ln(B/V(D)}{T} = \frac{\ln(500,000/481,328.37)}{1} = 3.79\%$ for Bank 1, and for Bank 2, $YTM = \frac{\ln(800,000/720,634.55)}{1} = 10.45\%$. The credit risk premium for Bank 1 is $YTM_1 - r = 3.79\% - 3\% = .79\%$ and it is $YTM_2 - r = 10.45\% - 3\% = 7.45\%$ for Bank 2.