BAYLOR UNIVERSITY
HANKAMER SCHOOL OF BUSINESS
DEPARTMENT OF FINANCE, INSURANCE & REAL ESTATE

Risk Management
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Final Exam
Fall 2020

Name SOLUTIONS

Instructions:

This test consists of two sections:

1. The first section has five multiple-choice questions worth 5 points each. For the multiple-choice questions, circle the response that you think is the best answer for each question.
2. The second section has three problems worth 24 points each.

Good luck!

Section 1. Five Multiple Choice Questions (5 points each)

1. Harold is indifferent between \$2,500 for sure and a bet with a 60 percent chance of \$2,400 and a 40 percent chance of \$2,600. Harold is
   a. risk averse.
   b. risk loving.
   c. risk neutral.
   d. a profit maximizer.
   e. irrational.

> **Note:** Here, $E(W)$ = .6(2,400) + .4(2,600) = \$2,480. Since expected wealth (\$2,480) is less than the certainty equivalent of wealth (\$2,500), this implies that Harold is risk loving. If Harold were risk neutral, he would be indifferent between \$2,480 for sure and this bet. The fact that he is willing to pay more than \$2,480 implies risk loving behavior.

2. A risk-averse person has a utility function that, with wealth on the horizontal axis and utility on the vertical axis, as wealth increases,
   a. is a horizontal line.
   b. is a vertical line.
   c. has constant, positive slope.
   d. **is increasing at a decreasing rate**.
   e. is increasing at an increasing rate.

> **Note:** As we discussed in class, if one is risk averse, then one has *diminishing marginal utility*. Diminishing marginal utility has the property of utility increasing at a decreasing rate as wealth increases.

3. You pay \$3.75 to roll a normal die 1 time. You get \$1 for each dot that turns up. Your expected profit from this venture is
   a. -\$.75.
   b. -\$.25.
   c. \$.25.
   d. \$3.00.
   e. \$3.50.

> Note: $E(\pi) = \sum_{s=1}^{n} p_s X_s - 3.75 = (1/6)(1) + (1/6)(2) + (1/6)(3) + (1/6)(4)$
> $+ (1/6)(5) + (1/6)(6) - 3.75 = -\$.25.$

4. Expected utility is
   a. the profit from a given decision.
   b. a probability weighted average of possible profits.
   c. an evenly weighted average of possibility profits.
   d. a probability weighted average of possible utility levels.
   e. the expected profits plus a number that depends on risk.

> **Note:** $E(U(W)) = \sum_{s=1}^{n} p_s U(W_s)$, so response D is the best verbal description of the expected utility equation.

5. D.J. Trump is running for office with 500,000 voters who have pledged their support. To add more voters, he wants to choose n, the number of negative campaign ads to run, where $0 \le n \le 4$. The ads will backfire with probability $0.2n$ and give him no extra votes. Otherwise, the ads will not backfire (with probability $(1 - 0.2n)$) and give him $100{,}000 + 40{,}000n$ extra votes. So n = 0 implies a total of 600,000 votes. Assuming that D.J. Trump is *risk neutral*, he should choose to run ____ negative campaign ads (hint: round to the nearest integer value).
   a. 0.
   b. **1.**
   c. 2.
   d. 3.
   e. 4.

> **Note:** Since D.J. Trump is risk neutral, his objective is to select the number of negative campaign ads which maximizes the expected number of voters. Therefore, E(voters) = 500,000 + $[100{,}000+40{,}000n](1-.2n) = 600{,}000 + 20{,}000n - 8{,}000n^2$. Thus, dE(voters)$/dn$ = 20,000 – $16{,}000n = 0 \Rightarrow n = 20{,}000/16{,}000 = 1.25$. Since only integer values are possible, the optimal number of negative campaign ads to run is 1.

Section 2. Three problems (24 points each)

Problem 2.1 (24 points)
An entrepreneur has initial wealth of \$88. Her initial wealth is invested in two buildings, each of which is worth \$40. Her remaining \$8 in initial wealth is invested in cash. Each building has a 25% chance of being destroyed and a 75% chance of not suffering any damage. Because the buildings are located far away from each other, these risks are statistically independent.

Since the entrepreneur has \$8 in cash, she can use some or all of this money to purchase actuarially fair insurance policies to cover her risks. Note that the price for an actuarially fair insurance policy equals the expected value of the payoff (indemnity) provided by the insurance policy.

A. (6 points) Given the entrepreneur's cash resources, if she covers 60% of the first building's potential loss, what is the maximum level of coverage (in terms of proportion of potential loss) that she can purchase against the risk that the second building will be destroyed?

SOLUTION: The expected value of each building's potential loss is $E(L) = \sum_{s=1}^{n} p_s L_s = .25(40) =$ \$10. Since the price for an actuarially fair insurance policy equals the expected value of the payoff (indemnity) provided by the insurance policy, this means that the premium for the first building's insurance policy is $.6\sum_{s=1}^{n} p_s L_s = (.6).25(40) =$ \$6. Since the entrepreneur has \$8 in cash, the maximum she can pay to insure the second building is \$2, which implies a maximum level of coverage of 20%.

B. (6 points) Given the entrepreneur's cash resources, what is the maximum level of coverage (in terms of proportion of potential loss) for each building that will result in the same premium being paid for each policy?

SOLUTION: Since the most that the entrepreneur can spend on insurance is \$8 and the expected value of each building's potential loss is \$10, this means that she can cover 40% of the potential loss for each building for a premium of \$4 per building.

C. (6 points) Suppose the entrepreneur's utility function is $U(W) = \sqrt{W}$. Show that the entrepreneur is better off if she insures both buildings at the same level of coverage (for a total premium of \$8) than she would be if she implemented the risk management strategy implied in Part A of this problem.

SOLUTION: Since the risks are statistically independent, this implies that the joint probability distribution for both risks consists of 4 possible states of the world: 1) no loss on either building (with probability .75 x .75 = 56.25%), 2) losses on both buildings (with probability .25 x .25 = 6.25%), 3) loss on building 1 and no loss on building 2 (with probability .75 x .25 = 18.75%) and 4) no loss on building 1 and loss on building 2 (with probability .25 x .75 = 18.75%). Furthermore, we need to derive an equation for state-contingent wealth in each of these states. Under this scenario (where she covers 40% of the potential loss for each building for a premium of \$4 per building), this implies that state-contingent wealth is $W_s = W_0 - \alpha_1 p_1 - \alpha_2 p_2$

$-(1-\alpha_1)L_{1s}-(1-\alpha_2)L_{2s}=88-8-.6L_{1s}-.6L_{2s}$. Thus the following distribution of state-contingent wealth is implied by this equation:

State	p_s	L_{1s}	L_{2s}	W_s	$U(W_s)$
no loss on either building	56.25%	0	0	80	8.9443
losses on both buildings	6.25%	40	40	32	5.6569
loss on building 1 and no loss on building 2	18.75%	40	0	56	7.4833
no loss on building 1 and loss on building 2	18.75%	0	40	56	7.4833
	Expected Value:	**10**	**10**	**68**	**8.1909**

Thus the expected utility of this risk management decision is 8.1909. Now suppose that the entrepreneur implements the risk management decision implied in part A. In other words, she covers 60% of the first building's potential loss for a premium of $6 and 20% of the second building's potential loss for a premium of $2. This risk management decision results in the following distribution of state-contingent wealth:

State	p_s	L_{1s}	L_{2s}	W_s	$U(W_s)$
no loss on either building	56.25%	0	0	80	8.9443
losses on both buildings	6.25%	40	40	32	5.6569
loss on building 1 and no loss on building 2	18.75%	40	0	64	8.0000
no loss on building 1 and loss on building 2	18.75%	0	40	48	6.9282
	Expected Value:	**10**	**10**	**68**	**8.1837**

Thus the expected utility of this alternative risk management decision is 8.1837, which implies that she is better off if she insures both buildings at the same level of coverage.

D. (6 points) Explain *why* the expected utility of having the same level of coverage on both buildings is higher than the expected utility of having different levels of coverage.

SOLUTION: This raises an interesting question - why is the expected utility of having the same level of coverage on both buildings higher than the expected utility of having different levels of coverage? The answer is quite simple. We know from the expected utility theory that a mean preserving spread will always produce a lower expected utility ranking. In this problem, the alternative risk management decision involving different levels of coverage represents a mean preserving spread of the risk management decision involving the same level of coverage on both buildings. Looking closer, the source of the greater dispersion associated with the alternative risk management decision occurs whenever one building is destroyed and the other building doesn't suffer any damage. Comparing these tables, state-contingent wealth in the 3rd and 4th states varies when there are different levels of coverage, but is the same when the same level of coverage is selected for both buildings.

Problem 2.2 (24 points)

One of the industries most affected by oil prices is the airline industry. After labor costs, the price of jet fuel is typically the most important cost component for airline companies. In spite of this fact, hedging practices vary significantly within the industry. For example, Southeast Airlines (LOV) has more than half of its total jet fuel exposure hedged at the equivalent of $30-$40 per barrel of oil, whereas United States Airlines (USA) does not hedge any of its jet fuel exposure.

Currently, the market value of United States Airlines' assets is $26 billion, and it has promised to pay its bondholders $20 billion 1 year from now. Southeast Airlines holds assets worth $7 billion, and it has promised to pay its bondholders $3.5 billion1 year from now. The riskless rate of interest is 5% per year, the standard deviation of the return on USA's assets is 50% per year, and the standard deviation of the return on LOV's assets is 30% per year.

A. (6 points) What is the total market value of USA debt? What is the yield to maturity for USA debt?

SOLUTION: The total market value of USA debt is equal to the difference between the value of a safe bond with the same promised payment and the value of a limited liability put option; i.e.,

$$V(D_{USA}) = B_{USA}e^{-rT} - V(Max(B_{USA} - V(F_{USA}))$$

The value of USA's limited liability put option, $V(Max(B_{USA} - V(F_{USA}))$, is calculated as follows:

$$V(Max(B_{USA} - V(F_{USA})) = B_{USA}e^{-rT}N(-d_2) - V(F_{USA})N(-d_2).$$

Next, we calculate d_1 and d_2; $d_1 = \frac{\ln(V(F)/B) + (r + .5\sigma^2)T}{\sigma\sqrt{T}} = \frac{\ln(26/20) + (.05 + .5(.5^2))1}{.5\sqrt{1}} =$ .8747, and $d_2 = d_1 - \sigma\sqrt{T} = 0.8747 - .5\sqrt{1} = .3747$. Therefore, $N(-d_1) = 19.09\%$, $N(-d_2) = 35.39\%$, and the value of the USA debt is:

$$\begin{aligned} V(D_{USA}) &= e^{-.05}20 \text{ billion } - (e^{-.05}20 \text{ billion}(.3539) - 26 \text{ billion}(.1909)) \\ &= \$19,024,588,490 - \$1,771,013,941 = \$17,253,574,549. \end{aligned}$$

Since USA has promised to pay its bondholders $20 billion and the market value of its debt is $17.253 billion, this means that

$$\$20 \text{ billion} = e^{YTM(T)}\$17.253 \text{ billion} \Rightarrow YTM = \ln(20/17.253) = 14.77\%.$$

B. (6 points) What is the total market value of LOV debt? What is the yield to maturity for LOV debt?

SOLUTION: Applying the same solution procedure as in part A of this problem,

$$d_1 = \frac{\ln(V(F)/B) + (r + .5\sigma^2)T}{\sigma\sqrt{T}} = \frac{\ln(7/3.5) + (.05 + .5(.3^2))1}{.3\sqrt{1}} = 2.6272, \; d_2 = d_1 - \sigma\sqrt{T} =$$

$2.6272 - .3\sqrt{1} = 2.3272$. Therefore, $N(-d_1) = 0.43\%$, $N(-d_2) = 1\%$, and the value of the LOV debt is:

$$V(D_{LOV}) = e^{-.05}3.5 \text{ billion } - (e^{-.05}3.5 \text{ billion}(.01) - 7 \text{ billion}(.0043))$$
$$= \$3,329,302,986 - \$3,085,742 = \$3,326,217,244.$$

Since LOV has promised to pay its bondholders \$3.5 billion and the market value of its debt is \$3.326 billion, this means that

$$\$3.5 \text{ billion} = e^{YTM(T)}\$3.326 \text{ billion} \Rightarrow YTM = \ln(3.5/3.326) = 5.09\%.$$

Now suppose that the federal government initiates a loan guarantee program that requires taxpayers to fully guarantee USA and LOV bonds against the risk of default.

C. (6 points) What is the dollar value of the federal government's loan guarantee to USA bondholders? What effect does the initiation of the federal government's loan guarantee to USA have upon the yield to maturity for USA debt?

SOLUTION: The dollar value of the federal government's loan guarantee to USA bondholders corresponds to the value of USA's limited liability put option, which is \$1,771,013,941 (as calculated in part A above). Since the fully guaranteed value of USA debt is equal to \$19,024,588,490, this means that the yield to maturity is $YTM = \ln(20/17.024) = 5\%$.

D. (6 points) What is the dollar value of the federal government's loan guarantee to LOV bondholders? What effect does the initiation of the federal government's loan guarantee to LOV have upon the yield to maturity for LOV debt?

SOLUTION: Using similar logic for LOV as we did for USA, we find that the dollar value of the federal government's loan guarantee corresponds to the value of LOV's limited liability put option, which is \$3,085,742 (as calculated in part B above). Since the fully guaranteed value of LOV debt is equal to \$3,329,302,986, this means that the yield to maturity is $YTM = \ln(3.5/3.329) = 5\%$.

Problem 2.3 (24 points)

Suppose that the (pre-loss and pre-tax) value of your company one period from now will be $600. However, your company's assets are subject to the following loss distribution:

L_s	Probability
$600	25%
$400	25%
$200	25%
$0	25%

The government assesses a tax rate of 50% on asset values exceeding $300, and a 0% tax rate whenever asset values fall below this amount. Assume that investors are risk neutral and the interest rate is 0 percent. Furthermore, assume that any insurance premiums paid are fully tax deductible, as are uninsured losses.

A. (6 points) Suppose that your company may fully insure this risk at an actuarially fair price. What would be the after-tax value of your company if you decided not to purchase insurance? What would be its value if you purchased insurance?

SOLUTION: The actuarially fair price for insurance is $E(L)$ = .25(600) + .25(400) + .25(200) = $300. By purchasing insurance, you guarantee that your firm will not pay any tax, since this will guarantee that the net value of the firm will always be $600 - $300 = $300. However, if you are uninsured, you will pay taxes when losses are $200 or less. The following table shows the calculation of after-tax firm value when insurance is not purchased:

				Uninsured
Pre-loss Earnings	**L(s)**	**Earnings(s)**	**Probability (p(s))**	**After Tax Earnings**
$600	$600	$0	25%	$0
$600	$400	$200	25%	$200
$600	$200	$400	25%	$350
$600	$0	$600	25%	$450
Expected Value	**$300**	**$300**		**$250**

B. (6 points) What is the net present value of purchasing insurance?

SOLUTION: The net present value of purchasing insurance is the difference in after-tax firm value when you are insured versus uninsured; i.e., $300 - $250 = $50.

C. (6 points) Do you recommend purchasing actuarially fair insurance? Why or why not?

SOLUTION: You should purchase actuarially fair insurance because by doing so you increase the value of the firm's shares by $50.

D. (6 points) What would be your recommendation about buying insurance if the premium loading on this policy was 20 percent? Be sure to justify your answer.

SOLUTION: With a 20 percent loading, insurance now costs $60 more, so you should not insure in this case because buying insurance will cause firm value to be $240, compared with $250 if no insurance is purchased.