Finance 4335 Final Exam Formula Sheet

I. Expected Utility

$$E\left(U(W)\right) = \sum_{s=1}^{n} p_s U(W_s).$$

II. Demand for Insurance

State-Contingent Wealth: $W_s = W_0 - E(I)(1+\lambda) - L_{u,s}$, where

- W_0 = initial wealth;
- $E(I)$ = expected value of the indemnity;
- λ = % premium loading (note: insurance is actuarially fair if $\lambda = 0$);
- $E(I)(1+\lambda)$ = price of insurance, also known as the "insurance premium"; and
- $L_{u,s}$ = the uninsured loss (note: under full coverage, $L_{u,s} = 0$.

III. Put-call Parity Equation (assuming no dividends; options are European):

$$c + Ke^{-rn\delta t} = p + S,$$

where
δt = the length of one timestep;
n = the number of timesteps until expiration;
$T = n\delta t$ = time until expiration ($T = 1$ corresponds to 1 year);
c = value of a call option with an exercise price of K and n timesteps until expiration;
p = value of a put option with an exercise price of K and n timesteps until expiration;
r = the annualized riskless rate of interest; and
S = the value of the underlying asset.

IV. Risk Neutral Probability of an "up" move:

$$q = \frac{e^{r\delta t} - d}{u - d},$$

- where q = the risk neutral probability of an "up" move; u = 1 plus the rate of return from an "up" move; and d = 1 plus the rate of return from a "down" move.

V. Risk Neutral Valuation Formula for pricing a 1-timestep European call or put option on a non-dividend paying stock:

$$f = e^{-r\delta t}[qf_u + (1-q)f_d],$$

- where f_u = the value of the option at the u node; f_d = the value of the option at the d node, and f = the current arbitrage-free price of the option.

VI. Risk Neutral Valuation Formula for pricing an n-timestep European call option on a non-dividend paying stock; also known as the Cox-Ross-Rubinstein (CRR) Option Pricing Formula:

$$c = e^{-rn\delta t}\left[\sum_{j=a}^{n}\left(\frac{n!}{j!\,(n-j)!}\right)q^j(1-q)^{n-j}\left(u^j d^{n-j}S - K\right)\right],$$

- where j corresponds to the number of up moves which occur during n timesteps, and $a =$ the smallest integer value $> \ln(K/Sd^n)/\ln(u/d)$.
- Thus, a indicates the *minimum* number of up moves required in order for a call option to expire in-the-money, and $n-a$ corresponds to the *maximum* number of down moves required in order for an otherwise identical put option to expire in-the-money.

VII. Black-Scholes-Merton European Option Pricing Formulas:

$$c = SN(d_1) - e^{-rT}KN(d_2) \text{ and } p = e^{-rT}KN(-d_2) - SN(-d_1),$$

where

$d_1 = \dfrac{\ln(S/K) + (r + .5\sigma^2)T}{\sigma\sqrt{T}}$;

$d_2 = d_1 - \sigma\sqrt{T}$;

$\sigma =$ annualized volatility of underlying asset's rate of return;

$T =$ time until expiration ($T = 1$ corresponds to 1 year);

$N(d_1) =$ standard normal distribution evaluated at d_1;

$N(d_2) =$ standard normal distribution evaluated at d_2;

$N(-d_1) = 1 - N(d_1) =$ standard normal distribution evaluated at $-d_1$; and

$N(-d_2) = 1 - N(d_2) =$ standard normal distribution evaluated at $-d_2$.

VIII. Credit Risk:

- Value of Risky Debt $V(D)$: $V(D) = Be^{-r(T)} - V[\max(0, B-F)]$
- Value of Limited Liability Put $V[\max(0, B-F)]$: Determined by applying the Black-Scholes-Merton pricing formula for the value of a put option with underlying asset value $V(F)$ and exercise price B, where B is the promised payment to the firm's creditors;
- Yield to Maturity (YTM): Suppose $B = V(D)e^{YTM(T)}$. Then $YTM = \dfrac{\ln(B/V(D))}{T}$.
- Credit Risk Premium: Credit Risk Premium $= YTM - r$.

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