Finance 4335 (Spring 2021) Midterm 1 Synopsis

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February 22, 2021

I. The overarching Finance 4335 concept in the first part of this course centers around the notion that people vary in terms of their preferences for bearing risk. Although we focused most of our attention upon modeling risk averse behavior, we also considered examples of risk neutrality (where you only care about expected wealth and are indifferent about riskiness of wealth) and risk loving (where you actually *prefer* to bear risk and are willing to pay money for the opportunity to do so).

II. Regardless of whether one is risk averse, risk neutral, or risk loving, the foundation for decision-making under conditions of risk and uncertainty is expected utility. Given a choice amongst various risky alternatives, one selects the choice that has the highest utility ranking.

   A. If you are risk averse, then $E(W) > W_{CE}$ and the difference between $E(W)$ and W_{CE} is equal to the risk premium λ. Some practical applications – if you are risk averse, then you are okay with buying "expensive" insurance at a price that exceeds the expected value of payment provided by the insurer, since (other things equal) you would prefer to transfer risk to someone else if it is not too expensive to do so. On the other hand, you are not willing to pay more than the certainty equivalent of wealth for a bet on a sporting event or a game of chance such as rolling dice or tossing a coin.

   B. If you are risk neutral, then $E(W) = W_{CE}$ and $\lambda = 0$; risk is inconsequential and all you care about is maximizing the expected value of wealth.

   C. If you are risk loving, then $E(W) < W_{CE}$ and $\lambda < 0$; i.e., you are quite happy to pay for the opportunity to (on average) lose money. This is because risk is, by definition, a "*sought-after*" attribute for someone with risk loving preferences.

III. We also discussed a couple of different methods for calculating λ.

   A. The so-called "exact method" involves calculating expected utility ($E(U(W))$), setting expected utility equal to the certainty-equivalent of wealth ($E(U(W)) = U(W_{CE})$), and solving for W_{CE} directly; e.g., if $E(U(W)) = U(W_{CE}) = 10$ and $U(W_{CE}) = \sqrt{W_{CE}}$, then $W_{CE} = 100$; if $E(W) = \$110$, then the risk premium $\lambda = E(W) - W_{CE} = \$10$.

   B. The "approximate method" involves solving directly for λ by evaluating the Arrow-Pratt coefficient ($R_A(W) = -U''/U'$) at the expected value of wealth and multiplying it by half of the variance of wealth; i.e., for $U(W) = \sqrt{W}, \lambda \cong .5\sigma_W^2 R_A(E(W)) =$

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$.5(4,400)(.5/\$110) = \$10$, and for $U(W) = \ln W$, $\lambda \cong .5\sigma_W^2 R_A(E(W)) = .5(4,400)(1/\$110)$ $= \$20$. This method provides important intuitive insights into the determinants of risk premiums. Specifically, we find that risk premiums depend upon two factors: 1) the magnitude of the risk itself (as indicated by variance), and 2) the degree to which the decision-maker is risk averse (as indicated by the Arrow-Pratt coefficient).

IV. We talked about "special cases" of expected utility – specifically, the mean-variance and the stochastic dominance models. If we impose various restrictive assumptions upon expected utility, then these models emerge as "special cases".

A. As long as the various restrictive assumptions required by these models apply, we can be confident that if risk X "dominates" risk Y, then the expected utility for X is greater than the expected utility for Y; a result which obtains for *all arbitrarily risk averse decision-makers.*

B. Of these two models, the mean-variance model is more restrictive than stochastic dominance. Indeed, the mean-variance model is *not* an appropriate method for risk evaluation under a variety of circumstances. For example, if one risk has a *higher mean and variance* than another risk, then we need further information about the decision-maker's utility function in order to determine which risk is preferred; just knowing the mean and variance is *not sufficient* in such a case.

C. Furthermore, the mean-variance model implicitly assumes that *risks are symmetrically distributed and have "thin" tails*; examples of such distributions include the binomial distribution in the discrete setting and the normal distribution in the continuous setting. However, if the underlying distribution is skewed or fat-tailed, then it is *not appropriate* to rank-order risks based upon the mean-variance framework, because variance only partially captures risk. To illustrate this, we considered a numerical example (see pp. 5-7 of the Decision Making Under Risk and Uncertainty (Part 4) lecture note) in which a positively skewed risk with a lower mean and a higher variance has higher expected utility than a symmetrically distributed risk with higher mean and lower variance.

D. Stochastic dominance appears to be more "robust" than the mean-variance model, because (unlike the mean-variance model) the stochastic dominance model can accommodate broader risk characteristics such as skewness and kurtosis.