Baylor University
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Risk Management
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Midterm Exam 2
Sample Midterm Exam 2

Name: SOLUTIONS

Notes:

1. Please read the instructions carefully.
2. This test consists of four problems, but you are only required to complete three out of four problems. At your option, you may complete all four problems, in which case I will count the three highest scoring problems for your exam grade.
3. Since the number of points only adds up to 96, students who complete this exam will also receive an additional 4 points. Thus, the maximum number of points possible for this exam is 100.

Good luck!

Problem 1 (32 points)

An entrepreneur has initial wealth of \$88. Her initial wealth is invested in two buildings, each of which is worth \$40. Her remaining \$8 in initial wealth is invested in cash. Each building has a 25% chance of being destroyed and a 75% chance of not suffering any damage. Because the buildings are located far away from each other, these risks are statistically independent.

Since the entrepreneur has \$8 in cash, she can use some or all of this money to purchase actuarially fair insurance policies to cover her risks. Note that the price for an actuarially fair insurance policy equals the expected value of the payoff (indemnity) provided by the insurance policy.

1. (8 points) Given the entrepreneur's cash resources, if she covers 60% of the first building's potential loss, what is the maximum level of coverage (in terms of proportion of potential loss) that she can purchase against the risk that the second building will be destroyed?

   SOLUTION: The expected value of each building's potential loss is $E(L) = \sum_{s=1}^{n} p_s L_s = .25(40) =$ \$10. Since the price for an actuarially fair insurance policy equals the expected value of the payoff (indemnity) provided by the insurance policy, this means that the premium for the first building's insurance policy is $.6 \sum_{s=1}^{n} p_s L_s = (.6).25(40) = \$6$. Since the entrepreneur has \$8 in cash, the maximum she can pay to insure the second building is \$2, which implies a maximum level of coverage of 20%.

2. (8 points) Given the entrepreneur's cash resources, what is the maximum level of coverage (in terms of proportion of potential loss) for each building that will result in the same premium being paid for each policy?

SOLUTION: Since the most that the entrepreneur can spend on insurance is \$8 and the expected value of each building's potential loss is \$10, this means that she can cover 40% of the potential loss for each building for a premium of \$4 per building.

3. (8 points) Suppose the entrepreneur's utility function is $U(W) = \sqrt{W}$. Show that the entrepreneur is better off if she insures both buildings at the same level of coverage (for a total premium of \$8) than she would be if she implemented the risk management strategy implied in Part A of this problem.

   SOLUTION: Since the risks are statistically independent, this implies that the joint probability distribution for both risks consists of 4 possible states of the world: 1) no loss on either building (with probability .75 x .75 = 56.25%), 2) losses on both buildings (with probability .25 x .25 = 6.25%), 3) loss on building 1 and no loss on building 2 (with probability .75 x .25 = 18.75%) and 4) no loss on building 1 and loss on building 2 (with probability .25 x .75 = 18.75%). Furthermore, we need to derive an equation for state-contingent wealth in each of these states. Under this scenario (where she covers 40% of the potential loss for each building for a premium of \$4 per building), this implies that state-contingent wealth is $W_s = W_0 - \alpha_1 p_1 - \alpha_2 p_2 - (1-\alpha_1)L_{1s} - (1-\alpha_2)L_{2s} = 88 - 8 - .6L_{1s} - .6L_{2s}$. Thus, the following distribution of state-contingent wealth is implied by this equation:

State	p_s	L_{1s}	L_{2s}	W_s	$U(W_s)$
no loss on either building	56.25%	0	0	80	8.9443
losses on both buildings	6.25%	40	40	32	5.6569
loss on building 1 and no loss on building 2	18.75%	40	0	56	7.4833
no loss on building 1 and loss on building 2	18.75%	0	40	56	7.4833
	Expected Value:	**10**	**10**	**68**	**8.1909**

Thus, the expected utility of this risk management decision is 8.1909. Now suppose that the entrepreneur implements the risk management decision implied in part A. In other words, she covers 60% of the first building's potential loss for a premium of $6 and 20% of the second building's potential loss for a premium of $2. This risk management decision results in the following distribution of state-contingent wealth:

State	p_s	L_{1s}	L_{2s}	W_s	$U(W_s)$
no loss on either building	56.25%	0	0	80	8.9443
losses on both buildings	6.25%	40	40	32	5.6569
loss on building 1 and no loss on building 2	18.75%	40	0	64	8.0000
no loss on building 1 and loss on building 2	18.75%	0	40	48	6.9282
	Expected Value:	**10**	**10**	**68**	**8.1837**

Thus the expected utility of this alternative risk management decision is 8.1837, which implies that she is better off if she insures both buildings at the same level of coverage.

4. (8 points) Explain *why* the expected utility of having the same level of coverage on both buildings is higher than the expected utility of having different levels of coverage.

SOLUTION: This raises an interesting question - why is the expected utility of having the same level of coverage on both buildings higher than the expected utility of having different levels of coverage? The answer is quite simple. We know from the expected utility theory that a mean preserving spread will always produce a lower expected utility ranking. In this problem, the alternative risk management decision involving different levels of coverage represents a mean preserving spread of the risk management decision involving the same level of coverage on both buildings. Looking closer, the source of the greater dispersion associated with the alternative risk management decision occurs whenever one building is destroyed and the other building doesn't suffer any damage. Comparing these tables, state-contingent wealth in the 3^{rd} and 4^{th} states varies when there are different levels of coverage, but is the same when the same level of coverage is selected for both buildings.

Problem 2 (32 points)

Consider an economy with three kinds of drivers: safe, careless, and crazy. There is an equal number of each of these drivers, and each driver has initial wealth of \$150 and utility $U(W) = \sqrt{W}$. In any given year, a safe driver gets into an accident with probability $p_s = 0.1$; a careless driver gets into an accident with probability $p_d = 0.2$, and a crazy driver gets into an accident with probability $p_c = 0.3$. An accident leads to repair costs of \$50, and has no other consequences.

Suppose that an insurance company offers full coverage insurance policies to these drivers; i.e., when accidents occur, claim payments of \$50 are made which fully cover repair costs. The drivers decide whether to purchase these full coverage policies.

A. (8 points) Suppose the insurance company can distinguish between the three types of drivers, and offers each driver an actuarially fair full coverage insurance policy based on his or her accident probability. Which of the drivers will buy such a policy?

SOLUTION: If insurance is actuarially fair, we know from the Bernoulli hypothesis that risk averse policyholders will prefer full coverage.

Simply invoking the Bernoulli hypothesis is sufficient for earning full credit. However, it is also acceptable if the student demonstrates that the safe, careless, and crazy drivers all have higher expected utility when insurance is actuarially fair. Actuarially fair insurance for the safe drivers costs $E(L_s) = \sum_{s=1}^{n} p_{s,s} L_s = .1(50) = \$5$. For careless drivers, actuarially fair insurance costs $E(L_d) = \sum_{s=1}^{n} p_{s,d} L_s = .2(50) = \$10$, and for crazy drivers, actuarially fair insurance costs $E(L_c) = \sum_{s=1}^{n} p_{s,c} L_s = .3(50) = \$15$.

Next, we compute expected utility when there is no insurance:

- EU (uninsured safe driver): $E(U(W_s)) = \sum_{s=1}^{n} p_{s,s} U(W_s) = .1\sqrt{100} + .9\sqrt{150} = 12.0227.$
- EU (uninsured careless driver): $E(U(W_d)) = \sum_{s=1}^{n} p_{s,d} U(W_s) = .2\sqrt{100} + .8\sqrt{150} = 11.7980.$
- EU (uninsured crazy driver): $E(U(W_c)) = \sum_{s=1}^{n} p_{s,c} U(W_s) = .3\sqrt{100} + .7\sqrt{150} = 11.5732.$

Expected utility is always higher when these drivers can fully insure at actuarially fair prices:

- EU (insured safe driver): $E(U(W_s)) = \sqrt{145} = 12.0416.$
- EU (insured careless driver): $E(U(W_d)) = \sqrt{140} = 11.8322.$
- EU (insured crazy driver): $E(U(W_c)) = \sqrt{135} = 11.6190.$

B. (8 points) Now suppose that although the insurance company cannot distinguish between the three types of drivers, it decides to offer to offer a full coverage insurance policy to all drivers for a price of \$10. Show that safe drivers are not willing to purchase such a policy, whereas careless and crazy drivers will purchase such a policy.

SOLUTION: If a full coverage policy costs \$10, this means that the expected utility of full coverage is $\sqrt{140} = 11.8322$ for everyone. Since safe drivers have higher expected utility when they remain uninsured, they will not be willing to purchase this policy. We have already determined that the careless drivers will insure at a price of \$10, so we know that they will stay in the market. The crazy drivers are certainly quite happy about this deal; they would have insured at a price of \$15 but are now getting an even better deal. However, now the insurer will lose money because the crazy drivers have an expected cost of \$15 and the insurer is only collecting \$10 in premium from each of these drivers.

C. (8 points) Since safe drivers are not willing to purchase the policy described in part B, the insurance company decides to increase the price of the policy, hoping to not lose money. This time, it offers a full coverage insurance policy for a price of \$15. Show that safe and careless drivers are not willing to purchase such a policy, whereas crazy drivers will purchase such a policy.

SOLUTION: If a full coverage policy costs \$15, this means that the expected utility of full coverage is $\sqrt{135} = 11.619$ for the careless and crazy drivers. Since careless drivers have higher expected utility when they remain uninsured, they will not be willing to purchase this policy. However, since \$15 is an actuarially fair price for crazy drivers, they will purchase coverage. The "equilibrium" here is one in which only the crazy drivers purchase and insurance, whereas the safe and careless drivers remain uninsured.

D. (8 points) This problem illustrates how adverse selection can limit insurability; here, even though safe and careless drivers are risk averse and would certainly be interested in purchasing coverage, the presence of the crazy drivers makes this challenging. Explain how offering partial coverage to the safe and careless drivers can solve this problem.

SOLUTION: The basic dilemma here is that when the insurer tries to charge an average premium, then the low risk insureds drop out of the risk pool, which in turn raises the average cost of providing insurance for the remaining policyholders. The insurer gets stuck in a vicious cycle; when she raises premiums in response to the lower risk policyholders dropping out, this in turn aggravates the problem even more. This process continues until only the worst risks are left. However, offering appropriately priced *partial* coverage policies to safe drivers and careless drivers solves the adverse selection problem by incentivizing self selection, such that the safe and crazy drivers are motivated to purchase partial coverage policies which are designed and priced to appeal to them and not to crazy drivers.

Problem 3 (32 points)

Suppose that you are interested in investing in a portfolio consisting of stocks and bonds. You have already decided to invest in an equity fund that is indexed to the S&P 500, but are not sure whether to invest in Government bonds or Catastrophe ("Cat") bonds. The Cat bonds are riskier than Government bonds because Cat bond investors must forfeit interest and principal payments if a Category 4 or 5 hurricane hits Florida. The following table summarizes the expected returns and standard deviations for these securities:

Security	**Expected Return**	**Standard Deviation**
Cat Bonds	8%	15%
Equity Fund	12%	20%
Gov't Bonds	5%	8%

The correlation between Cat Bond returns and Equity Fund returns is -.1, whereas the correlation between Government Bond returns and Equity Fund returns is .10.

A. (10 points) Suppose you would like to earn 10% on your portfolio. What is the standard deviation of a portfolio consisting of Cat Bonds and the Equity Fund that would have an expected return of 10%?

<u>**Solution:**</u> $E(r_p) = \sum_{i=1}^{n} w_i E(r_i) = w_1 8\% + (1 - w_1)12\% = 10\% \Rightarrow w_1 = .50$. Therefore, $\sigma_p = \sqrt{w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1w_2\rho_{12}\sigma_1\sigma_2} = \sqrt{.50^2\left(.15^2\right) + .50^2\left(.20^2\right) + 2\,(.5)\,(.5)\,(-.1)\,(.15)\,(.2)} = 11.88\%$.

B. (10 points) What is the standard deviation of a portfolio consisting of Government Bonds and the Equity Fund that would have an expected return of 10%?

Solution: $E(r_p) = \sum_{i=1}^{n} w_i E(r_i) = w_1 5\% + (1 - w_1) 12\% = 10\% \Rightarrow w_1 = 2/7$. Therefore, $\sigma_p = \sqrt{w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1w_2\rho_{12}\sigma_1\sigma_2} = \sqrt{\left(\frac{2}{7}\right)^2 \left(.08^2\right) + \left(\frac{5}{7}\right)^2 \left(.20^2\right) + 2\left(\frac{2}{7}\right)\left(\frac{5}{7}\right)(.10)(.08)(.2)} = 14.69\%$.

C. (12 points) Suppose the riskless rate of interest is 3%, and you can borrow or lend against either of the portfolio combinations described in parts A and B above. Which of these two portfolio combinations is preferred by arbitrarily risk averse investors? Justify your answer.

Solution: The Sharpe ratio for the portfolio described in part A is equal to $\frac{E(r_p) - r_f}{\sigma_p} = \frac{.10 - .03}{.1188} = .589$, whereas the Sharpe ratio for the portfolio described in part B is equal to $\frac{E(r_p) - r_f}{\sigma_p} = \frac{.10 - .03}{.1469} = .477$. Since the Sharpe ratio for the part A portfolio is greater than for the part B portfolio, arbitrarily risk averse investors will prefer to borrow and lend against the part A portfolio.

Problem 4 (32 points)

Suppose that you are considering investing in a portfolio consisting of two securities, A and B. You have estimated that these two securities will provide the following set of statecontingent returns ($r_{A,s}$ and $r_{B,s}$), depending on how well or poorly the economy performs (note: p_s represents the probability that state s will occur):

State of Economy	p_s	$r_{A,s}$	$r_{B,s}$
Boom	50.00%	5.00%	40.00%
Bust	50.00%	10.00%	-5.00%

1. (10 points) What are the expected returns for securities A and B?

   Solution:

$$E(r_A) = \sum_{s=1}^{n} p_s r_{As} = .5(.05) + .5(.10) = 7.5\%.$$

$$E(r_B) = \sum_{s=1}^{n} p_s r_{Bs} = .5(.40) + .5(-.05) = 17.5\%.$$

2. (10 points) What are the standard deviations of the returns for securities A and B?

   Solution:

$$\sigma_A^2 = \sum_{s=1}^{n} p_s (r_{As} - E(r_A))^2 = .5(.05 - .075)^2 + .5(.10 - .075)^2 = .0625\%; \text{ therefore, } \sigma_A = 2.50\%;$$

$$\sigma_B^2 = \sum_{s=1}^{n} p_s(r_{Bs} - E(r_B))^2 = .5(.40 - .175)^2 + .5(-.05 - .175)^2 = 5.0625\%; \text{ therefore, } \sigma_B = 22.50\%;$$

Note: since there are equal probabilities of Boom and Bust, the standard deviation of each security can also be determined by simply calculating half of the total dispersion between the Boom and Bust states; thus, σ_A= 0.5(.10 - .05) = 2.5%, and σ_B= 0.5(.40 + .05) = 22.5%.

3. (12 points) Suppose that state contingent returns on the risk free asset ($r_{f,s}$) and the market portfolio ($r_{m,s}$) are estimated as follows:

State of Economy	p_s	$r_{f,s}$	$r_{m,s}$
Boom	50.00%	5.00%	25.00%
Bust	50.00%	5.00%	0.00%

Using this information in conjunction with the information in Problem 4 on Securities A and B, are Securities A and B underpriced, overpriced, or correctly priced? Justify your answer.

Solution: In order to determine whether Securities A and B are underpriced, overpriced, or correctly priced, we need to calculate the beta coefficients for each security. Since $\beta_i = \sigma_{im}/\sigma_m^2$, this requires calculating the covariances between A and the market and B and the market, and then dividing each of these numbers through by the variance of the market.

First, we find the variance of the market:

$$\sigma_m^2 = \sum_{i=1}^{n} p_s(r_{ms} - E(r_m))^2 = .5(.25 - .125)^2 + .5(0 - .125)^2 = [1/2\,(.25 - .0)]^2 = 1.5625\%.$$

Next, we find the covariances:

$$\begin{aligned}\sigma_{Am} &= \sum_{s=1}^{n} p_s(r_{As} - E(r_A))(r_{ms} - E(r_m)) \\ &= .5\,(.05 - .075)\,(.25 - .125) + .5\,(.10 - .075)\,(0 - .125) \;= -.0031\end{aligned}$$

$$\begin{aligned}\sigma_{Bm} &= \sum_{s=1}^{n} p_s(r_{Bs} - E(r_B))(r_{ms} - E(r_m)) \\ &= .5\,(.40 - .175)\,(.25 - .125) + .5\,(-.05 - .175)\,(0 - .125) = .0281\end{aligned}$$

Therefore, $\beta_A = \dfrac{\sigma_{Am}}{\sigma_m^2} = \dfrac{-.0031}{.015625} = -.2$, and $\beta_B = \dfrac{\sigma_{Bm}}{\sigma_m^2} = \dfrac{.0281}{.015625} = 1.80$.

Consequently,

$$E(r_A) = r_f + [E(r_m) - r_f]\beta_A = 5\% + [12.5\% - 5\%]\,(-.20) = 3.5\%, \text{and}$$
$$E(r_B) = r_f + [E(r_m) - r_f]\beta_B = 5\% + [12.5\% - 5\%]\,(1.80) = 18.5\%.$$

Since security A should only provide an expected return of 3.5% but is currently priced to provide an expected return of 7.5%, this means that this security is underpriced. Investors will realize this, and consequently there will be excess demand for security A, which will cause its price to increase and its expected return to fall until it is in line with the CAPM. Conversely, security B should provide an expected return of 18.5% but is currently priced to provide an expected return of only 17.5%, which means that this security is overpriced. Investors will realize this, and consequently there will be excess supply for security B, which will cause its price to decrease and its expected return to rise until it is in line with the CAPM.