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Risk Management
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Problem Set #10

Name: SOLUTIONS

Problem 1.

Suppose that the (pre-loss and pre-tax) value of your company one period from now will be \$600. However, your company's assets are subject to the following loss distribution:

L_s	Probability
\$600	25%
\$400	25%
\$200	25%
\$0	25%

The government assesses a tax rate of 50% on asset values exceeding \$300, and a 0% tax rate whenever asset values fall below this amount. Assume that investors are risk neutral and the interest rate is 0 percent. Furthermore, assume that any insurance premiums paid are fully tax deductible, as are uninsured losses.

A. Suppose that your company may fully insure this risk at an actuarially fair price. What would be the after-tax value of your company if you decided not to purchase insurance? What would be its value if you purchased insurance?

SOLUTION: The actuarially fair price for insurance is $E(L) = .25(600) + .25(400) + .25(200) = \$300$. By purchasing insurance, you guarantee that your firm will not pay any tax, since this will guarantee that the net value of the firm will always be \$600 - \$300 = \$300. However, if you are uninsured, you will pay taxes when losses are \$200 or less. The following table shows the calculation of after-tax firm value when insurance is not purchased:

				Uninsured
Pre-loss Earnings	**L(s)**	**Earnings(s)**	**Probability (p(s))**	**After Tax Earnings**
\$600	\$600	\$0	25%	\$0
\$600	\$400	\$200	25%	\$200
\$600	\$200	\$400	25%	\$350
\$600	\$0	\$600	25%	\$450
Expected Value	**\$300**	**\$300**		**\$250**

B. What is the net present value of purchasing insurance?

SOLUTION: The net present value of purchasing insurance is the difference in after-tax firm value when you are insured versus uninsured; i.e., \$300 - \$250 = \$50.

C. Do you recommend purchasing actuarially fair insurance? Why or why not?

SOLUTION: You should purchase actuarially fair insurance because by doing so you increase the value of the firm's shares by \$50.

D. What would be your recommendation about buying insurance if the premium loading on this policy was 20 percent? Be sure to justify your answer.

SOLUTION: With a 20 percent loading, insurance now costs \$60 more, so you should not insure in this case because buying insurance will cause firm value to be \$240, compared with \$250 if no insurance is purchased.

Problem 2.

Suppose that shareholders are contemplating making an investment today (at t=0) and are considering different financing alternatives. The payoffs on this investment occur one period from today (at t=1). At t=1, only two states of the world (loss and no loss) may occur with equal probabilities. If the loss occurs, the investment will be worth \$2,000, and if there is no loss, then the firm will be worth \$4,000. However, the firm has an option to rebuild the asset at a cost of \$1,600 should a loss occur. Assume that shareholders are risk neutral, the interest rate is zero and bankruptcy is costless.

A. What is the net present value of rebuilding the asset?

SOLUTION: Given this problem's assumptions, the NPV equals the expected value of the net cash flow benefit from rebuilding. Since a rebuilding cost of \$1,600 occurs when the firm suffers a \$2,000 loss, this means that the NPV is therefore equal to .5(\$2,000-\$1,600) = \$200. We can also compute NPV by comparing the value of an all equity firm before and after the investment is made. Let Π = income before loss, L_s = state contingent loss, and I_s = state contingent building cost. Furthermore, let $V(E_{r,s})$ represent the state contingent value of equity when there is rebuilding, and $V(E_{n,s})$ represent the state contingent value of equity when there is no rebuilding. Then NPV = $E[V(E_{r,s})]$ – $E[V(E_{n,s})]$, where $E[V(E_{r,s})]$ - $E[V(E_{n,s})] = \sum_{s=1}^{n} p_s V(E_{r,s})$- $\sum_{s=1}^{n} p_s V(E_{n,s})$.

Therefore,

$E[V(E_{r,s})] = \sum_{s=1}^{n} p_s(\Pi - I(s))$ = .5(4,000-0) + .5(4,000-1,600) = \$3,200, and

$E[V(E_{n,s})] = \sum_{s=1}^{n} p_s(\Pi - L(s))$ = .5(4,000-0) + .5(4,000-2,000) = \$3,000; consequently,

NPV = E[$V(E_{r,s})$] - E[$V(E_{n,s})$] = \$3,200 - \$3,000 = \$200.

B. Suppose the firm is all equity financed. Will shareholders rebuild the asset in the event of a loss? Why or why not?

SOLUTION: As we showed in part A above, by rebuilding the asset we make ourselves \$200 richer, so we will rebuild the asset in the event of a loss.

C. Suppose that as an alternative to equity financing, shareholders can issue zero coupon bonds. If the promised payment on the bonds equals \$2,000, will shareholders rebuild the asset in the event of a loss? Why or why not?

SOLUTION: We know that underinvestment can occur when there is default risk. However, with a \$2,000 promised payment on the bonds, there is no possibility of default, irrespective of whether the loss occurs. Therefore, bonds are worth \$2,000, and the value of equity is \$1,200 if the asset is rebuilt, compared with \$1,000 if the asset is not rebuilt. Consequently, shareholders will rebuild the asset in the event of a loss, because to do so makes them \$200 richer.

D. Suppose shareholders issue zero coupon bonds and promise to repay \$3,000 at t=1. With this type of financing arrangement, will shareholders rebuild the asset in the event of a loss? Why or why not?

SOLUTION: Now, we introduce the possibility of default, so things may change, since the benefits of the investment will accrue to bondholders rather than shareholders. Specifically, let E[$V(B_{n,s})$] be the value of the firm's bonds if there is underinvestment, and E[$V(B_{r,s})$] equal the value of the firm's bonds if rebuilding occurs:

$E[V(B_{n,s})]=\sum_{s=1}^{n} p_s\{Min(B, \Pi - L(s))\}$ = .5(Min(\$3,000, \$4,000) + .5(Min(\$3,000, \$4,000-\$2,000) = .5(\$3,000) + .5(\$2,000) = \$2,500, and

$E[V(B_{r,s})]=\sum_{s=1}^{n} p_s\{Min(B, \Pi - I(s))\}$ =.5(Min(\$3,000, \$4,000) + .5(Min(\$3,000, \$4,000-\$1,600) = .5(\$3,000) + .5(\$2,400) = \$2,700.

Since the total value of the firm is \$3,000 when there is underinvestment and \$3,200 otherwise, this implies that the value of the firm's shares is \$500 no matter what happens. However, shareholders are not indifferent about rebuilding. In the present case, the prospect of default creates a moral hazard. Shareholders would like to convince bondholders that they will rebuild in the event of a loss, collect higher proceeds from issuing bonds, and renege on their promise. Since bondholders are rational and recognize the incentives for shareholders to employ such a tactic, they will assume the worst; viz., that shareholders will not rebuild. Consequently, bonds will be priced at \$2,500, which guarantees that shareholders will not rebuild (since bondholders get all the benefits and shareholders do not have anything to gain from rebuilding).

E. Suppose that instead of issuing zero coupon bonds with a promised repayment of \$3,000, shareholders decide to issue zero coupon bonds with a promised repayment of \$2,600 and purchase an actuarially fair insurance policy with a deductible of \$1,400. With this type of financing arrangement, will shareholders rebuild the asset in the event of a loss? Why or why not?

SOLUTION: By altering the face value of debt and purchasing insurance, the firm makes bonds completely safe, so the market value of bonds must be $2,600. We know this occurs because the net cash flow in the loss state will be equal to $4,000 less the rebuilding cost of $1,600 plus the payment from the insurer of $200, or $2,600. However, by introducing insurance, shareholder incentives change. Since the payoff on the firm's equity will be $0 when the loss occurs and $1,400 otherwise, this means that shares will be worth $700 if rebuilding occurs and $500 otherwise. In other words, shareholders appropriate the full net present value from rebuilding. Therefore, shareholders will rebuild the asset.