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Risk Management
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Problem Set 5

Name: SOLUTIONS

Assume a consumer has initial wealth of $10,000, utility of wealth $U(W) = \sqrt{W}$, and her wealth is subject to the following loss distribution:

Loss Amount (L_s)	Probability (p_s)
$0	50%
$1,000	25%
$9,000	25%

A. Determine the expected utility of wealth, assuming the consumer is uninsured.

SOLUTION: In order to determine the expected utility of wealth for this consumer, we must calculate the state contingent value of wealth $W_s = W_0 - L_s$, and then use this information to determine the state contingent value of the utility of wealth $U(W_s)$:

p_s	L_s	$p_s L_s$	W_s	$p_s W_s$	$U(W_s)$	$p_s U(W_s)$
50%	$0	$0	$10,000	$5,000	100	50
25%	$1,000	$250	$9,000	$2,250	94.87	23.72
25%	$9,000	$2,250	$1,000	$250	31.62	7.91
	$E(L)$	$2,500	$E(W)$	$7,500	$E(U(W))$	81.62

Therefore, expected utility is 81.62.

B. Calculate the actuarially fair price for a full (100%) coverage insurance policy.

SOLUTION: The actuarially fair price for full coverage insurance is simply the expected value of loss, which we calculated in the above table as $2,500.

C. Show that it is optimal for this consumer to purchase a full coverage insurance policy at its actuarially fair price by comparing expected utility in the absence of insurance with expected utility in the presence of insurance.

SOLUTION: In order to determine whether this consumer would purchase full insurance at the actuarially fair price, we must calculate the expected utility of full insurance. Since the actuarially fair price is $2,500, by fully insuring this consumer will have certain wealth of $7,500. The utility of $7,500 is $U(W) = \sqrt{7500} = 86.60$. Since the expected utility of full insurance is greater than the expected utility of being uninsured, this consumer will prefer to be fully insured. Of course, this is simply a formal numerical proof of the Bernoulli hypothesis, which states that arbitrarily risk averse consumers will always fully insure at actuarially fair prices.

D. If only full coverage insurance policies are available in the market, what is the maximum price that this consumer is willing to pay for such a policy?

SOLUTION: To answer this question, we must compute the certainty equivalent of wealth. Since the expected utility of remaining uninsured is 81.62, this means that the certainty equivalent of wealth is $81.62^2 = \$6,662.28$. Therefore, the maximum premium that this consumer is willing to pay is $\$10,000 - \$6,662.28 = \$3,337.72$.

E. Suppose this consumer may choose one of the following four risk management strategies:

1) Policy A fully covers all losses for a price of $3,125;
2) Policy B has a $1,000 deductible and costs $2,500;
3) Policy C covers 80% of all losses for a price of $2,500; and
4) Self-insure.

Which of these four strategies will this consumer choose? Explain why.

SOLUTION: We begin by calculating state-contingent wealth under the four alternatives of self-insurance, Policy A, Policy B, or Policy C:

State Contingent Wealth

p_s	L_s	Self-insure	Policy A	Policy B	Policy C
50%	$0	$10,000	$6,875	$7,500	$7,500
25%	$1,000	$9,000	$6,875	$6,500	$7,300
25%	$9,000	$1,000	$6,875	$6,500	$5,700
E(.)	**$2,500**	**$7,500**	**$6,875**	**$7,000**	**$7,000**

As we can see from the above table, the highest expected wealth is obtained from self-insuring, which should be obvious because this is the only way to escape a premium loading. In order to determine which policy should be purchased, we must calculate the *expected utility* associated with these four alternatives:

State Contingent Wealth

p_s	L_s	Self-insure	Policy A	Policy B	Policy C
50%	$0	100.00	82.92	86.60	86.60
25%	$1,000	94.87	82.92	80.62	85.44
25%	$9,000	31.62	82.92	80.62	75.50
Expected Utility		81.62	82.92	83.61	83.54

Since Policy B has the highest expected utility, we would expect the consumer to purchase that policy.