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Risk Management
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Problem Set 6

Name: SOLUTIONS

Problem #1 (50 points)

Suppose you own a company which you expect will earn the following set of state-contingent profits, *prior* to compensating your company's manager:

	Weak Economy Profit (π_{weak})	**Average Economy Profit** ($\pi_{average}$)	**Good Economy Profit** (π_{good})	**Expected Profit** ($E(\pi)$)
p_s	30%	40%	30%	
Low Effort π_s	\$5,000,000	\$10,000,000	\$15,000,000	\$10,000,000
High Effort π_s	\$7,000,000	\$12,000,000	\$17,000,000	\$12,000,000

As shown here, state-contingent profits depend jointly on the state of the economy *and* the manager's effort level. Specifically, if the manager selects a low effort level, then your business is less profitable than when the manager selects a high effort level; specifically, $E(\pi_{\text{low effort}})$ = \$10 million compared with $E(\pi_{\text{high effort}})$ = \$12 million.

The manager's utility $U(W) = \begin{cases} \sqrt{W} \text{ under low effort} \\ \sqrt{W} - 100 \text{ under high effort} \end{cases}$, and her initial wealth is $W_0 = \$0$. Therefore, "minus 100" indicates the *disutility* associated with expending extra effort.

As the risk neutral owner of this company, you wish to maximize expected profit, net of the cost of the manager's compensation. You are considering three mutually exclusive compensation schemes:

- Compensation Scheme #1: a fixed salary of \$575,000;
- Compensation Scheme #2: a payment of 6 percent of profits; or
- Compensation Scheme #3: a salary of \$500,000 plus half of any profits above \$15 million.

Calculate expected profit, net of the cost of the manager's compensation, for each of these compensation schemes. Which compensation scheme will you choose, and why?

SOLUTION: The solution requires that we calculate the expected cost of compensation and utility under each of the three compensation schemes, given low and high effort by the manager. In the case of a fixed salary of \$575,000, the manager will optimally choose low effort, since low effort utility is $U(W) = \sqrt{575,000} = 758.29$ whereas high effort utility is $U(W) - 100 = \sqrt{575,000} - 100 = 658.29$. Given the manager's choice of low effort under the fixed salary scheme, expected profit (after compensating the manager) would be $E(\pi_{\text{low effort}}) - E(\text{Scheme \#1 Compensation}) = \$10,000,000 - \$575,000 = \$9,425,000$.

Since Compensation Schemes #2 and #3 directly tie state-contingent compensation to state-contingent profit, we need to consider both low and high effort by the manager for those cases. In the table below are calculations of state-contingent compensation and utility under Compensation Schemes #2 and #3:

State-Contingent Compensation and Utility under Compensation Schemes #2 and #3	**Weak Economy π**	**Average Economy π**	**Good Economy π**	**Expected Value**
p_s	30%	40%	30%	
Compensation Under Scheme #2 (low effort)	\$300,000	\$600,000	\$900,000	\$600,000
Compensation Under Scheme #2 (high effort)	\$420,000	\$720,000	\$1,020,000	\$720,000
Compensation Under Scheme #3 (low effort)	\$500,000	\$500,000	\$500,000	\$500,000
Compensation Under Scheme #3 (high effort)	\$500,000	\$500,000	\$1,500,000	\$800,000
Utility Under Scheme #2 (low effort)	547.72	774.60	948.68	758.76
Utility Under Scheme #2 (high effort)	548.07	748.53	909.95	736.82
Utility Under Scheme #3 (low effort)	707.11	707.11	707.11	707.11
Utility Under Scheme #3 (high effort)	607.11	607.11	1,124.74	762.40

In the case of Compensation Scheme #2, the manager's expected utility under low effort is 758.76, compared with expected utility under high effort of 736.82. Therefore, if Compensation Scheme #2 is implemented, then the manager would respond by choosing low effort, and expected profit (net of managerial compensation) under Compensation Scheme #2 would be $E(\pi_{\text{low effort}}) - E(\text{Scheme \#2 Compensation}) = \$10,000,000 - \$600,000 = \$9,400,000$.

Finally, in the case of Compensation Scheme #3, expected utility of manager with low effort is 707.11, and with high effort, expected utility is 762.40. Therefore, if Compensation Scheme #3 is implemented, then the manager would respond by choosing high effort, and expected profit (net of managerial compensation) under Compensation Scheme #3 would be $E(\pi_{\text{high effort}}) - E(\text{Scheme \#3 Compensation}) = \$12,000,000 - \$800,000 = \$11,200,000.$

Problem #2 (50 points)

Assume that all drivers are risk averse with utility $U(W) = \sqrt{W}$. Each driver has cash in the amount of \$300 and owns a car worth \$1,000 (thus initial wealth $W_0 = \$1,300$). However, drivers have different probabilities of crashing their cars; some are high risk ($p_H = 30\%$), some are medium risk ($p_M = 20\%$), and others are low risk ($p_L = 10\%$). There are only two states of the world, crash and no crash. In the crash state, drivers suffer a total loss; i.e., cars become worthless whenever crashes occur.

Insurance is available, although it is not compulsory. Thus, drivers insure themselves only if the expected utility of being insured exceeds the expected utility of going without insurance.

While insurers know that there are equal numbers of high, medium, and low risk drivers, there is asymmetric information; specifically, insurers cannot identify *which* drivers are high, medium, and low risk.

A. Gecko Insurance Company is a monopolist; it has no competitors, so insurance can only be obtained from Gecko. Gecko offers full coverage ($\alpha = 1$) insurance policies for \$200. Which drivers purchase policies at this price, and which drivers go without insurance? What is Gecko Insurance Company's average profit (or loss) per policy sold?

   SOLUTION: Since the expected losses for low, medium and high risk drivers are \$100, \$200, and \$300 respectively, this implies that this policy is actuarially fair for medium risk drivers. We know from the Bernoulli principle that whenever insurance is actuarially fair, a risk averse decision-maker prefers to fully insure. Furthermore, this price represents a \$100 discount from the actuarially fair price for high risk drivers, so it also follows that high-risk drivers will purchase this policy. Finally, from the viewpoint of low risk drivers, a \$200 price represents an implied premium loading of 100%, and we know from Mossin's theorem that a risk averse decision-maker prefers to partially insure at actuarially unfair prices. Given that the choice is binary; i.e. insure completely or not at all, this means that Gecko will not be able to sell policies to low risk drivers (that low risk drivers drop out is easily confirmed; note that $E(U(W)) = .1\sqrt{300} + .9\sqrt{1300} = 34.18$ for uninsured low risk drivers, and only $E(U(W)) = \sqrt{1,100} = 33.17$ for insured drivers). Of course, by dropping out this causes Gecko to lose on average \$50 per policy sold, since the expected profit per medium risk driver is \$0 whereas the expected loss per high risk driver is \$100.

B. Suppose Gecko Insurance Company raises the price for full coverage insurance policies from \$200 to \$250. At this price, which drivers purchase policies and which drivers go without insurance? What is Gecko Insurance Company's average profit (or loss) per policy sold, given this increase in price?

   SOLUTION: From our discussion in part A, since a \$300 price provides high risk drivers with greater utility than self-insuring, they will also purchase such policies when the price is \$250. Furthermore, since insurance was already unattractive to low risk drivers at a price of \$200, this price increase makes insurance even more unattractive, so the low risk drivers will remain uninsured. Next, we check whether medium risk drivers will purchase insurance. Expected utility from buying insurance is $E(U(W)) = \sqrt{1,050} = 32.40$, compared with expected utility from forgoing coverage of $E(U(W)) =$

$.2\sqrt{300} + .8\sqrt{1,300} = 32.31$. Thus, medium risk drivers will purchase full coverage policies for \$250. Since the fair price for medium risk drivers is \$200 compared with the fair price for high risk drivers of \$300, this means that Gecko will lose \$50 per high risk driver and gain \$50 per medium risk driver. Since there are equal numbers of high and medium risk drivers, the average profit per policy sold is $(\$50 + (-\$50))/2 = \$0$.

C. Suppose a new insurance company (MostStates Insurance Company) is formed for the purpose of challenging Gecko Insurance Company's monopoly. MostStates offers three different policies: 1) a full coverage ($\alpha = 1$) policy for \$300, 2) a partial coverage ($\alpha = .3$) policy for \$60, and 3) a partial coverage ($\alpha = .1$) policy for \$10. Which policies offered by MostStates and Gecko (if any) will high, medium, and low risk drivers select in this more competitive environment?

SOLUTION: From part B, we know that high risk drivers will prefer Gecko's \$250 full coverage policy to MostStates's \$300 full coverage policy, so they will stick with Gecko's \$250 full coverage policy. However, since a \$250 full coverage price represents a 25 percent premium loading for medium risk drivers and a 150 percent premium loading for low risk drivers, we need to compare expected utility for Gecko's \$250 full coverage policy (which is $E(U(W)) = \sqrt{1,050} = 32.4$) compared with the expected utilities of MostStates's partial insurance policies for medium and low risk drivers:

- Medium risk drivers (($\alpha = .3$) policy): $E(U(W)) = .2\sqrt{540} + .8\sqrt{1,240} = 32.82$.
- Medium risk drivers (($\alpha = .1$) policy): $E(U(W)) = .2\sqrt{390} + .8\sqrt{1,290} = 32.68$.
- Low risk drivers (($\alpha = .3$) policy): $E(U(W)) = .1\sqrt{540} + .9\sqrt{1,240} = 34.02$.
- Low risk drivers (($\alpha = .1$) policy): $E(U(W)) = .1\sqrt{390} + .9\sqrt{1,290} = 34.3$.

By inspection, MostStates's $\alpha = .3$ policy provides higher expected utility (32.82) than all other insurance policies and more than the expected utility of remaining uninsured ($E(U(W)) = .2\sqrt{300} + .8\sqrt{1,300} = 32.31$) for medium risk drivers. Similarly, we find that MostStates's $\alpha = .1$ policy provides higher expected utility (34.3) than all other insurance policies and more than the expected utility of remaining uninsured ($E(U(W)) = .1\sqrt{300} + .9\sqrt{1,300} = 34.18$) for low risk drivers.

D. (4 points) What impact will MostStates's entry into the insurance market have upon the average profit (or loss) per policy sold by Gecko Insurance Company?

SOLUTION: Since Gecko loses all medium risk drivers to MostStates (and they never could attract any of the low risk drivers), this means that Gecko will on average lose \$50 per policy.

E. (4 points) What is the average profit (or loss) per policy sold by MostStates Insurance Company?

SOLUTION: By inspection, MostStates's $\alpha = .3$ policy represents actuarially fair coverage for medium risk drivers; the \$60 price represent 30 percent of the full coverage actuarially fair price for medium risk drivers of \$200. Similarly, MostStates's $\alpha = .1$ policy represents actuarially fair coverage for low risk drivers; the \$10 price represent

10 percent of the full coverage actuarially fair price for low risk drivers of \$100. Since MostStates only sells partial coverage, actuarially fair policies, this implies that the average profit per policy sold by MostStates Insurance Company is \$0.