# Baylor University
# Hankamer School of Business
# Department of Finance, Insurance & Real Estate

Risk Management
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Problem Set 8

Name: SOLUTIONS

**Problem 1** (40 points)

Suppose the current value of a (non-dividend-paying) stock is $1,000, and the annual continuously compounded riskless rate of interest is 3%. Based on the example provided on pp. 9-13 from the "Derivatives Theory, Part 1" lecture note, solve parts A and B below.

A. (20 points)What is the "arbitrage-free" price for a forward contract on this stock which matures 1 year from today?

SOLUTION: $F = Se^{rT} = \$1,000e^{.03} = \$1,030.45$

B. (20 points) Suppose the forward price is $1,040. Describe a profitable zero risk, zero net investment trading strategy involving the forward contract and its replicating portfolio. If you implement such a strategy, how much profit will you earn?

SOLUTION: Since the forward price of $1,040 exceeds its "arbitrage-free" price by $9.55, it is too expensive. In order to take advantage of this mispricing in a way that involves zero risk and zero net investment, one should sell a 1-year forward contract for $1,040, buy the stock for $1,000, and fund the stock purchase with $1,000 in (riskless) debt. The following table succinctly summarizes this trading strategy:

| **Transaction** | **Payoff now** | **Payoff @ *T*** |
|---|---|---|
| Sell Forward | $0 | $1,040 - $S_T$ |
| Buy Stock | ($1,000) | $S_T$ |
| Borrow | $1,000 | ($1,000)$e^{.03}$ = ($1,030.45) |
| **Arbitrage Profit** | **$0** | **$9.55** |

**Problem 2** (60 points)

The price of a share of Zoom stock is currently $50. It is known that at the end of 1 year, the Zoom share price will be either $62.50 or $40. The riskless interest rate is 2% per year.

A. (10 points) Calculate the price of a 1-year European call option on Zoom stock with an exercise price of $50 by applying the replicating portfolio approach.

SOLUTION: According to the replicating portfolio approach:

$$\begin{aligned}
C_u &= Max(0, S_u - K) = 12.50 \\
C_d &= Max(0, S_d - K) = 0 \\
V_{RP} &= \Delta S + B \\
\Delta S &= \frac{C_u - C_d}{S(u-d)} S = \frac{12.50}{50(.45)} 50 = .5556(50) = 27.78 \\
B &= \frac{uC_d - dC_u}{e^{r\delta t}(u-d)} = \frac{.25(0) - .2(12.50)}{e^{.02(1)}(.45)} = -21.78 \\
\therefore C = V_{RP} &= \Delta S + B = \$27.78 - \$21.78 = \$6.00.
\end{aligned}$$

B. (10 points) Calculate the price of a 1-year European call option on Zoom stock with an exercise price of \$50 by applying the delta hedging approach.

SOLUTION: According to the Delta Hedging Approach:

$$\begin{aligned}
V_H &= C - \Delta S = C - \Delta 50. \\
V_H^u &= V_H^d \Rightarrow 12.50 - \Delta 62.50 = 0 - \Delta 40 \Rightarrow \Delta = .5556. \\
V_H^u &= V_H^d = -22.22 \\
V_H &= C - \Delta 50 = C - 27.78 = -e^{-.02} 22.22 = -.9802(22.22) = -21.78 \\
\therefore C &= \$6.00
\end{aligned}$$

C. (10 points) Calculate the price of a 1-year European call option on Zoom stock with an exercise price of \$50 by applying the risk neutral valuation approach.

SOLUTION: The risk neutral probability of an up move is $q = \frac{e^{r\delta t} - d}{u - d} = \frac{e^{.02} - .8}{1.25 - .8} = .4893$. Since the stock is worth $\$50(1.25) = \$62.50$ at the $u$ node and $\$50(.8) = \$40$ at the $d$ node, this means that the call is only in the money at the $u$ node; specifically, it is worth \$12.50 at that node. Therefore, the price of a one-year call option is

$$C = e^{-r\delta t}[qC_u + (1-q)C_d] = e^{-.02}[.4893(12.50)] = \$6.00.$$

D. (10 points) Calculate the price of a 1-year European put option on Zoom stock with an exercise price of \$50.

SOLUTION: According to the put-call parity equation, $C + Ke^{-r\delta t} = P + S$; therefore, $P = C + Ke^{-r\delta t} - S \Rightarrow P = 6 + 50e^{-.02(1)} - 50 = \$5.01$.

E. (20 points) Next, add another 1-year timestep; i.e., it is known that at the end of 2 years, the Zoom share price will be \$78.13, \$50, or \$32. Calculate the price of a 2-year European call option on Zoom stock with an exercise price of \$50. Also calculate the price of a 2-year European put option on Zoom stock with an exercise price of \$50.

SOLUTION: Note that the call option is only in the money at the $uu$ node, where it is worth \$28.13. It is worthless at the $ud$ and $dd$ nodes. Applying risk neutral valuation,

the price of a two-year call option is

$$\begin{aligned} C &= e^{-2r\delta t}[q^2 C_{uu} + 2q(1-q)C_{ud} + (1-q)^2 C_{dd}] \\ &= e^{-.04}[.4893^2(28.13)] = \$6.47. \end{aligned}$$

Regarding the (otherwise identical) 2-year put option, put-call parity indicates that the put is worth \$4.51:

$$P = C + Ke^{-2r\delta t} - S = 6.47 + \$50e^{-.04} - \$50 = \$4.51.$$