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Risk Management — Name: SOLUTIONS
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Problem Set 9

Two firms exist that are identical in all respects except for the risk of their assets. Both firms have assets worth $1,000,000, and have issued zero coupon bonds with a face value of $500,000. The standard deviation of the return on these firm 1's assets is 30%, and the standard deviation of the return on firm 2's assets is 50%. Assume that both firms will be liquidated one year from today and that the rate of interest is 5%.

1. What is the fair market value for the bonds issued by firm 1? What is the dollar value of its limited liability put option? What is the yield to maturity on its bonds?

   SOLUTION: The fair market value for the bonds issued by firm 1 is equal to the fair market value for riskless bonds, minus the fair market value for the limited liability put option. Therefore, we must compute the value of firm 1's equity, and since the fair market value for the bonds plus the value equity must equal $1,000,000, we can back into the value of the bonds. Furthermore, since the value of the limited liability put option is equal to the difference between the value of safe bonds and risky bonds that both have face values of $500,000, we can also back into the value of the limited liability put option.

   The value of firm 1's equity can be computed by calculating d_1 and d_2, then $N(d_1)$ and $N(d_2)$, and then combining these probability measures with the current value of assets and the current value of safe bonds:

$$d_1 = \frac{\ln(S_0/B) + (r + .5\sigma^2)T}{\sigma\sqrt{T}} = \frac{\ln(\$1\text{million}/\$.5\text{million}) + (.05 + .5(.09))}{.30} = 2.6272.$$

   Therefore, $d_2 = d_1 - \sigma\sqrt{T} = 2.6272 - .3 = 2.3272$. Consequently, $N(d_1) = 99.57\%$, $N(d_2) = 99.00\%$, and the value of equity is:

   $V_0^1(E) = V_0^1(F)N(d_1) - e^{-rT}BN(d_2) = \$1\text{ million}(.9957) - e^{-.05}(\$.5\text{ million})(.99) = \$524{,}826.11.$

   Since $V_0^1(E) = \$524{,}826.11$ and $V_0^1(F) = \$1$ million, this means that the market value of risky debt for firm 1, $V_0^1(D)$, is equal to $1 million - $524,826.11 = $475,173.89. Also, since the value of safe bonds is $e^{-.05}(\$.5\text{ million}) = \$475{,}614.71$, this means that the value of the limited liability put option is $475,614.71 - $475,173.89 = $440.82. Finally, the yield to maturity is the rate of return that will cause the risky bond value of $475,173.89 to grow to a maturity value of $500,000; i.e.,

$$500,000 = 475,173.89e^{YTM(T)};$$
$$500,000/475,173.89 = e^{YTM(1)};$$
$$YTM = \ln(500,000/475,173.89) = 5.09\%.$$

Solving directly for the limited liability put option,

$$V_0^1(Max(B-F,0) = e^{-rT}BN(-d_2) - V_0^1(F)N(-d_1)$$
$$= e^{-.05}(.5\text{million})(0.01) - (1\text{million})(0.0043) = 440.82.$$

2. What is the fair value for the bonds issued by firm 2? What is the dollar value of its limited liability put option? What is the yield to maturity on its bonds?

   SOLUTION: Following the same solution procedure as before, we find that for firm 2,

$$d_1 = \frac{\ln(S_0/B) + (r + .5\sigma^2)T}{\sigma\sqrt{T}} = \frac{\ln(\$1\text{million}/\$.5\text{million}) + (.05 + .5(.25))}{.50} = 1.7363.$$

   Therefore, $d_2 = d_1 - \sigma\sqrt{T} = 1.7363 - .5 = 1.2363$. Consequently, $N(d_1) = 95.87\%$, $N(d_2) = 89.18\%$, and the value of equity is:

   $V_0^2(E) = V_0^2(F)N(d_1) - e^{-rT}BN(d_2) =$ \$1 million(.9576) − $e^{-.05}$(\$.5 million)(.8918) = \$534,578.81.

   Since $V_0^2(E)=$ \$534,578.81 and $V_0^2(F) =$ \$1 million, this means that the market value of risky debt for firm 2, $V_0^2(D)$, is equal to \$1 million - \$534,578.81 = \$465,421.19. Also, since the value of safe bonds is $e^{-.05}$(\$.5 million) = \$475,614.71, this means that the value of the limited liability put option is \$475,614.71 - \$465,421.19 = \$10,193.52. Finally, the yield to maturity is the rate of return that will cause the risky bond value of \$465,421.19 to grow to a maturity value of \$500,000; i.e.,

$$500,000 = 465,421.19e^{YTM(T)};$$
$$500,000/465,421.19 = e^{YTM(1)};$$
$$YTM = \ln(500,000/465,421.19) = 7.17\%.$$

   Solving directly for the limited liability put option,

$$V_0^2(Max(B-F,0) = e^{-rT}BN(-d_2) - V_0^2(F)N(-d_1)$$
$$= e^{-.05}(.5\text{million})(0.1082) - (1\text{million})(0.0424) = 10,193.52.$$

3. Suppose an insurer offers the shareholders of both firms a credit enhancement scheme that will make their bonds riskless. What are the fair premiums for this insurance? What impact will this insurance have upon the yields to maturity of these bonds?

   SOLUTION: The fair premiums are equal to the values of the limited liability put option for the two firms; i.e., for firm 1 the fair premium is \$440.82, and for firm 2 the fair premium is \$10,193.52. By making the bonds of both firms riskless from the viewpoint of investors, the credit enhancement will cause the yields to maturity of the two bonds to be equal to the riskless rate of interest (5%).