

## Math, Probability, Statistics, and Finance Glossary

- **Algebra:** A generalization of arithmetic in which letters are used to represent numbers or quantities.
- **American Option:** An option that can be exercised any time until the option expires.
- **Annuity:** A stream of equal cash flow flows received at regularly spaced points in time.
- **Black-Scholes Formula:** A widely used formula that is used to price European put and call options as a function of the risk free rate, stock price, exercise price, duration of option, dividend payout rate, and stock's volatility.
- **Call Option:** An option that gives the owner the right to buy an asset (usually a stock) at a given price.
- **Cartesian Plane:** A system by which points in the plane are defined by two numbers: the point's x and y coordinates.
- **Comparative Statics:** The determination of how a formula changes when inputs to the formula are changed is known as comparative statics.
- **Complementary Events:** Two events are complementary events if they have no points in common and together include all points in an experiment's sample space.
- **Compounding of Interest: Continuous:** Interest (based on the sum of the initial investment and prior interest earned) is earned at every instant of time.
- **Compounding of Interest: Semi-Annual and Quarterly:** Interest (based on the sum of the initial investment and prior interest earned) is earned at periodic time intervals. If interest is earned semiannually, for example, then interest is received twice per year: at the beginning of the year and the middle of the year.
- **Concave function:** A function is concave for a value  $x$  if  $f''(x)$  (2nd derivative of  $x$ ) is less than or equal to 0. A function that is concave for all values of  $x$  (such as  $y = -x^2$ ) is referred to as a concave function. More intuitively, a function is concave if for all values of  $x$  the slope of the function is non-increasing.

- **Conditional Probability:** Given two events A and B the conditional probability of event A given that event B has occurred is written as  $P(A|B)$ . Intuitively, once we know that event B has occurred, this is the chance that event A will occur.
- **Continuous Random Variable:** A continuous random variable is used to describe an uncertain quantity (such as height, weight, or return on a stock) which can assume an infinite number of values and is defined over an interval or intervals of values.
- **Convex Function:** A function is convex for a value  $x$  if  $f''(x)$  (2nd derivative of  $x$ ) is greater than or equal to 0. A function that is convex for all values of  $x$  (such as  $y = x^2$ ) is referred to as a convex function. More intuitively, a function is convex if for all values of  $x$  the slope of the function is non-decreasing.
- **Correlation:** A unit-free measure of linear association between two data sets.
- **Covariance:** A measure of linear association between two data sets that depends on the units in which each data set is measured.
- **Cubic Function:** A polynomial of degree 3.
- **Degree:** The highest power of the variable that occurs in a polynomial.
- **Dependent Variable:** Often referred to as  $y$ , the dependent variable represents the value of a function that is computed based on the value(s) of the independent variable(s).
- **Differential Calculus:** A branch of mathematics concerned with determining the slope of a function at any point. Knowledge of differential calculus enables us to easily maximize and minimize functions and also graph complicated functions.
- **Discrete Random Variable:** A discrete random variable is used to describe an uncertain quantity (such as the roll of a die, number of cookies eaten in a day) that can only assume a finite number of values.
- **Efficient Market Hypothesis:** The theory, invented by University of Chicago finance professor Eugene Fama, that one cannot consistently achieve returns in excess of average market returns on a risk-adjusted basis, given the information publicly available at the time the investment is made.

- **Elasticity of Demand:** The percentage increase in demand that results from a 1% increase in price.
- **Equation:** A mathematical statement that expresses equality between two mathematical quantities.
- **European Option:** An option that can only be exercised on the option's expiration date.
- **Event:** A set of possible outcomes in a probabilistic experiment.
- **Expected Value of a Random Variable:** The average value you would expect to see of a random variable if you perform an experiment many times.
- **Expiration Date:** The last date on which an option can be exercised.
- **Face Value:** The amount of money (excluding coupons) paid to a bondholder on the bond's maturity date. The face value is usually a round number such as \$100 or \$1000.
- **First Derivative:** For a given value of  $x$  (say  $x_0$ ) the first derivative of  $f(x)$ , written as  $f'(x_0)$ , is the slope of the function  $f(x)$  when  $x = x_0$ .
- **Function:** A rule by which the numerical value of a variable(s) (the independent variable(s)) yields a numerical output value (the dependent variable).
- **Future Value:** Under the assumption of a given rate of return, future value measures the value of a cash flow moved forward in time. For example, if we earn 8% per year on our investment the future value of \$100 in two years would equal  $100(1.08)^2 = \$116.64$ .
- **Growing Perpetuity:** A sequence of periodic cash flows received for an infinite number of periods that grows at a constant rate per period. In finance, such an investment (e.g., a share of stock) can be valued by using the so-called "Gordon growth model", which is given by the equation  $P_0 = \frac{D_1}{r - g}$ , where  $P_0$  is the current price of the stock,  $D_1$  is the expected value of the dividend 1 year from now,  $r$  is the cost of equity for the company, and  $g$  is the constant growth rate for future dividend

payments. Note that if there is no growth (i.e.,  $g = 0$ ), then we have a level perpetuity; i.e.,  $P_0 = D_1 / r$ .

- **Hedging:** An investment strategy that reduces an individual's or an organization's risk. For example, buying put options on the market would hedge the risk a mutual fund faces that their investments in the market will drop in value.
- **Independent Events:** Two events are independent if the occurrence of one of the events does not change our estimate of the probability of the other event. In short, events A and B are independent if and only if  $P(A|B) = P(A)$ .
- **Independent Variable:** The number used as an input to a function of one variable. When we write  $y = f(x)$ , then x is the independent variable, and y is called the dependent variable because the value of y is determined by the independent variable x.
- **Inflection Point:** A point where a function changes from convex to concave or concave to convex. For example, for  $y = x^3$ ,  $x = 0$  is an inflection point because for  $x < 0$   $f(x)$  is concave and for  $f(x)$  is convex.
- **Inverse Function:** Usually we write  $y = f(x)$ . If we solve for x as a function of y then we can write  $x = g(y)$ . We say the function g is the inverse of the function f; e.g., if  $y = x$ , then  $x = 1/y$ , or if  $y = \ln x$ , then  $x = e^y$ .
- **Internal Rate of Return (IRR):** The rate of return that makes the Net present value (NPV) of a sequence of cash flows equal to 0. Some streams of cash flows have multiple IRR's while other streams of cash flows have no IRR.
- **Joint Probabilities:** Given two events A and B we define the joint probability of A and B to be the probability that events A and B both occur.
- **Mean:** The average of a set of numbers.
- **Median:** Given a set of numbers the median x is the number which has as many numbers in the set of numbers below x as above x. If there are  $2n + 1$  numbers in the set, the median is the  $n + 1$  largest number while if there are  $2n$  numbers in the set, the median is the average of the nth and  $n + 1$ st smallest numbers.

- **Mode:** Given a set of numbers, the most frequently occurring number in the set. If no number in the set appears more than once, then the mode does not exist.
- **Multivariate Function:** A function of more than one variable.
- **Mutually Exclusive Events:** A set of events are mutually exclusive if the occurrence of one event precludes the occurrence of any other event.
- **Natural Logarithm:**  $y$  is the natural logarithm of  $x$  (written  $y = \ln x$ ) if  $e^y = x$ . Here  $e$  is approximately 2.7182. For example,  $\ln e^4 = 4$ .
- **Negatively Skewed Histogram:** A histogram in which the data extends much further to the left of the most frequently occurring bin than to the right of the most frequently occurring bin.
- **Net Present Value (NPV):** The value of a sequence of cash flows expressed in today's dollars.
- **Normal Random Variable:** A continuous random variable that describes many quantities such as height, weight, or monthly sales of a product. A normal random variable is specified by its mean and standard deviation. The Excel function =NORMDIST( $x$ , mean, standard deviation, True) gives the probability that a normal random variable with a given mean and standard deviation is  $\leq x$ .
- **Outlier:** A point in a data set that seems "out of the ordinary." Most often a point is considered an outlier if it is more than 2 standard deviations away from the mean of the data set.
- **Parabola:** The graph of a function of the form  $y = a + bx + cx^2$ .
- **Parallel Lines:** Two lines are parallel if they have the same slope. Parallel lines never intersect.
- **Payback Criteria:** The amount of time needed for an investment to pay back its initial cash outflow.
- **PEMDAS:** A mnemonic device to help you remember the order of mathematical operations. Evaluate expressions within parentheses first, and then evaluate exponential terms, followed by multiplication and division terms and finally addition and subtraction terms.

- **Percentage:** Mathematical shorthand for 1/100. A percentage is a number expressed as fraction whose denominator is 100. Thus 1/4 is expressed as 25% because  $25/100 = 1/4$ .
- **Perpetuity:** An infinite sequence of equal periodic cash flows.
- **Polynomial:** A function of the form  $f(x) = a_0 + a_1x + a_2x^2 + \dots + a_nx^n$ .
- **Positively Skewed Histogram:** A histogram in which the data extends much further to the right of the most frequently occurring bin than to the left of the most frequently occurring bin.
- **Power Function:** A function of the form  $y = ax^b$ .
- **Probability density function (PDF):** A continuous random variable's pdf tells us the relative likelihoods of the random variable's possible values. The area under a pdf between a and b is the probability that the continuous random variable assumes a value between a and b.
- **Put Option:** An option that gives the owner the right to sell an asset (usually a stock) at a given price.
- **Quadratic Function:** A polynomial of degree 2.
- **Random Variable:** A function that associates a numerical value with every possible outcome of an experiment.
- **Sample Space:** The set of all possible outcomes for an experiment.
- **Sample Standard Deviation:** A measure of a data set's spread about its mean. It is computed as the square root of the data set's sample variance.
- **Sample Variance:** Given a set of data the sample variance is the average squared deviation of the data points from the sample mean. The sample variance measures the spread of a data set about its mean.
- **Second Derivative:** Written as  $f''(x)$ , a function's second derivative is the "slope of the slope." A positive second derivative indicates that the function's slope is

increasing while a negative second derivative indicates that a function's slope is decreasing.

- **Skewness:** A measure of a data set's asymmetry. A skewness value near 0 indicates a data set with a symmetric histogram. A skewness value greater than +1 indicates that a data set is positively skewed. A skewness value less than -1 indicates that a data set is negatively skewed.
- **Slope:** A straight line has the same slope everywhere; the line's slope is the change in  $y$  corresponding to a unit change in  $x$ . For any function, the function's first derivative  $f'(x_0)$  (if it exists) is the function's slope when  $x = x_0$ .
- **Slope Intercept Form of Line Equation:** A straight line with slope  $m$  and  $y$  intercept  $x = b$  may be written as  $y = mx + b$ .
- **Standard Deviation of Random Variable:** A measure of a random variable's spread about its mean. It is computed as the square root of the random variable's variance.
- **Strike Price:** The price at which a call option to buy or a put option to sell may be exercised; also commonly referred to as the exercise price.
- **Summation Notation:** Summation notation is shorthand for the addition of many terms.
- **Symmetric Histogram:** A histogram that looks the same to the left and right of the highest bar. For example, the most common IQ is 100 but nearly as many people have IQs near 80 as 120, etc.
- **Tangent Line:** A line that goes through a function's graph at a point and has the same slope as the function at that point.
- **Variable:** An unknown quantity which is represented by a letter (most often  $x$  or  $y$ ).
- **Variance of a Random Variable:** A measure of a random variable's spread about its mean defined as the average squared deviation of a random variable from its mean.
- **Volatility:** A measure of the spread or uncertainty in a stock's daily returns.
- **Zero Coupon Bond:** A bond that pays the owner a single payment of the date the bond matures.