

Stochastic Dominance Class Problem Solutions

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Consider two risky prospects, X_1 and X_2 , with payoffs given by:

$$X_{1,s} = \begin{cases} \$1 & \text{with probability } 50\% \\ \$9 & \text{with probability } 50\% \end{cases} \quad \text{and} \quad X_{2,s} = \begin{cases} \$4 & \text{with probability } 99\% \\ \$81 & \text{with probability } 1\% \end{cases}$$

Assume that your initial wealth (W_0) is \$0, and your utility $U(W) = \sqrt{W}$.

A. Does one investment first order stochastically dominate the other? Explain why or why not.

For there to be first order stochastic dominance, it must be the case that $F(W_{is}) \geq F(W_{js})$ for all s . In the table below, we check for first order stochastic dominance, and we find that there is no first order stochastic dominance, since when $W_s = 1$, $F(W_{1s}) > F(W_{2s})$, whereas when $W_s = 4$, $F(W_{1s}) < F(W_{2s})$.

$W_s = W_0 + X_s$	$f(W_{1s})$	$F(W_{1s})$	$f(W_{2s})$	$F(W_{2s})$
\$1	50.00%	50.00%	0.00%	0.00%
\$4	0.00%	50.00%	99.00%	99.00%
\$9	50.00%	100.00%	0.00%	99.00%
\$81	0.00%	100.00%	1.00%	100.00%

B. Compare these investments once again. Does one investment second order stochastically dominate the other? Explain why or why not.

Risk j second order dominates risk i if $\sum_{s=1}^n (F(W_{is}) - F(W_{js})) > 0$. Since $\sum_{s=1}^n (F(W_{1s}) - F(W_{2s})) = (.5 - 0) + (.5 - .99) + (1.0 - .99) + (1 - 1) = .02$, it follows that risk 2 second order dominates risk 1.

C. Calculate your expected utility ($E(U(W))$) for both investments.

$$E(U(W_1)) = .5(1) + .5(3) = 2$$

$$E(U(W_2)) = .99(2) + .01(9) = 2.07$$

D. Which investment should you choose? Explain why.

You should choose risk 2, since it second order dominates risk 1, which in turn implies that risk 2 also must also have higher expected utility than risk 1. This is true for virtually all arbitrarily risk averse utilities. For example, if $U(W) = \ln W$, then $E(U(W_1)) = .5(2.197) = 1.099$ and $E(U(W_2)) = .99(1.386) + .01(4.394) = 1.416$.