

## Midterm Exam #1 Formula Sheet

### 1. Expected value ( $E(W)$ ) and variance ( $\sigma_W^2$ )

- $E(W) = \sum_{s=1}^n p_s W_s$ , where  $p_s$  = the probability of state  $s$  and  $W_s$  = state  $s$  wealth, and
- $\sigma_W^2 = \sum_{s=1}^n p_s (W_s - E(W))^2$ .

### 2. Expected Utility ( $E(U(W))$ )

- $E(U(W)) = \sum_{s=1}^n p_s U(W_s)$ , where  $U(W_s)$  = state-contingent utility of wealth.

### 3. Certainty Equivalent of Wealth ( $W_{CE}$ ) and Risk Premium ( $\lambda$ ) - Two Methods

- “Exact” Method: Set  $E(U(W)) = U(W_{CE})$  and solve for  $W_{CE}$ ; then  $\lambda = E(W) - W_{CE}$ .
- Arrow-Pratt Method:  $\lambda = .5\sigma_W^2 R_A(E(W))$ , where  $R_A(W) = -U''(W)/U'(W)$ . Then  $W_{CE} = E(W) - \lambda$ .

### 4. Mean-Variance Model

If variance is a “complete” risk measure, then  $E(U(X_i)) > E(U(X_j))$  for all risk averse utility functions under the following set of conditions:

- $E(X_i) > E(X_j)$  and  $\sigma_{X_i} < \sigma_{X_j}$ ;
- $E(X_i) > E(X_j)$  and  $\sigma_{X_i} = \sigma_{X_j}$ ; and
- $E(X_i) = E(X_j)$  and  $\sigma_{X_i} < \sigma_{X_j}$ .

### 5. Stochastic Dominance Model

If  $X_i$  stochastically dominates  $X_j$ , then  $E(U(X_i)) > E(U(X_j))$  for all risk averse utility functions. Here are the formal definitions for first and second order stochastic dominance:

- First Order Stochastic Dominance: Investment  $i$  First Order Stochastic Dominates (FOSD) investment  $j$  if  $F(X_{j,s}) \geq F(X_{i,s})$  for all  $s$ .
- Second Order Stochastic Dominance: Investment  $i$  Second Order Stochastic Dominates (SOSD) investment  $j$  if  $\sum_{s=1}^n (F(X_{j,s}) - F(X_{i,s})) > 0$ .