

MORAL HAZARD CLASS PROBLEM SOLUTIONS

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Currently, the CEO receives a fixed salary of \$500,000. This salary represents her “reservation” salary; if the utility value of \$500,000 for certain is breached, then she will take a job elsewhere. The Board of Directors is concerned that she may be just “going through the motions” and would work harder if she were paid a bonus based upon the firm’s profits.

The CEO’s utility function is  $U = W^{0.5}$ , where state-contingent wealth  $W_s$  comprises initial wealth ( $W_0 = \$1,000,000$ ), salary ( $S$ ), bonus ( $B$ ), and cost of effort ( $C = \$50,000$ , which represents the opportunity cost of hard work for the CEO); thus,  $W_s = W_0 + S + B - C$ .  $C$  is only incurred by the CEO if she works hard. The firm’s owners are risk neutral, so they are interested in maximizing the expected value of profit after taking the CEO’s compensation into account.

The table below provides estimates of the firm’s profit (before CEO compensation is deducted) under three states of the world. The proposed new compensation plan would cut the CEO’s salary ( $S$ ) from \$500,000 to \$300,000 but give her a \$500,000 bonus if the firm’s profit (before CEO compensation is deducted) is greater than or equal to \$10,000,000.

	<b>Good Economy</b>	<b>Average Economy</b>	<b>Weak Economy</b>
	$p_s = 0.3$	$p_s = 0.3$	$p_s = 0.4$
<b>CEO Goes Through Motions</b>	\$10,000,000	\$8,000,000	\$6,000,000
<b>CEO Works Hard</b>	\$15,000,000	\$12,000,000	\$9,000,000

Note: see the spreadsheet at <http://fin4335.garven.com/execcomp.xls> for validation of the results that follow:

1. Under the current compensation scheme, is the Board correct in its assessment of the effort of the CEO? Why or why not?

**SOLUTION:** Under the current ( $S = \$500,000$ ,  $B = \$0$ ) compensation scheme, the CEO’s utility from going through the motions is:

$$EU_{GoThruMotions}^{Current} = (1,000,000 + 500,000)^{0.5} = (1,500,000)^{0.5} = 1,224.74.$$

This also represents her reservation level of utility. The CEO’s utility from working hard is:

$$EU_{WorkHard}^{Current} = (1,000,000 + 500,000 - 50,000)^{0.5} = (1,450,000)^{0.5} = 1,204.16$$

Since  $1,224.74 > 1,204.16$ , the CEO will go through the motions. The profit consequences under the current compensation scheme are as follows:

Going through the motions:

$$E(\pi_{GoThruMotions}) = 0.3*\$9,500,000 + 0.3*\$7,500,000 + 0.4*\$5,500,000 = \$7,300,000$$

The expected profit before bonus if the CEO works hard is:

$$E(\pi_{WorkHard}) = 0.3*\$14,500,000 + 0.3*\$11,500,000 + 0.4*\$8,500,000 = \$11,200,000$$

In other words, by paying a flat salary, the firm earns considerably less expected profit, since the CEO does not have the incentive to work hard. Thus the Board correct in its assessment of the effort of the CEO. The current compensation scheme is dysfunctional (not working)! She needs incentive pay, but the Board also has to take into consideration her employment alternatives; i.e., however the compensation package is parameterized, it must deliver no less than utility of 1,224.74 or she will switch firms.

2. Will the Board's new compensation scheme have its intended effect, i.e., will the CEO work hard?

**SOLUTION:** Under the proposed compensation scheme, the CEO's expected utility from going through the motions is:

$$EU_{GoThruMotions}^{Proposed} = 0.3(1,000,000 + 300,000 + 500,000)^{0.5} + 0.3(1,000,000 + 300,000)^{0.5} + 0.4(1,000,000 + 300,000)^{0.5} = 0.3(1,341.64) + 0.7(1,140.18) = 1,200.62$$

Under the proposed compensation scheme, the CEO's expected utility from working hard is

$$EU_{WorkHard}^{Proposed} = 0.3(1,000,000 + 300,000 + 500,000 - 50,000)^{0.5} + 0.3(1,000,000 + 300,000 + 500,000 - 50,000)^{0.5} + 0.4(1,000,000 + 300,000 - 50,000)^{0.5} = 0.6(1,322.88) + 0.4(1,118.03) = 1,240.94.$$

Therefore, the CEO will work hard under the proposed compensation scheme, since 1,240.94 > 1,200.62. This means that instead of only earning

$$E(\pi_{GoThruMotions}) = 0.3(10,000,000 - 300,000 - 500,000) + .3(8,000,000 - 300,000) + .4(6,000,000 - 300,000) = \$7,350,000, \text{ the firm can expect to earn}$$

$$E(\pi_{WorkHard}) = 0.3(15,000,000 - 300,000 - 500,000) + .3(12,000,000 - 300,000 - 500,000) + .4(9,000,000 - 300,000) = \$11,100,000.$$

So the new compensation scheme works for the CEO (she's better off than the previous compensation scheme—1,240.94 versus 1,224.74) and for the firm (expected profit is \$11,100,000 versus \$7,300,000). This occurs in spite of the fact that the proposed compensation scheme exposes the CEO to some risk; specifically, 40% of the time, she will work hard, earn a lower salary, and not earn any bonus at all due to a weak economy. She still earns more than her reservation level of utility.

3. What is the minimum level of bonus for the CEO in order for the Board's new compensation scheme to have its intended effect?

**SOLUTION:** Go to <http://fin4335.garven.com/execcomp.xls> and click on the "Salary plus Bonus" tab. This will take you to a worksheet which you can use in order to find the answer to this question. Specifically, you want to use Solver in order to determine the answer to this question. You'll find that a bonus of \$429,298 makes the CEO indifferent between working hard and leaving the firm. Theoretically, any amount more than \$429,298 will incentivize her to work hard. For example, in order to generate a 1/100 point increase in expected utility (i.e., have Solver determine the bonus amount that provides expected utility of 1,224.75 versus 1,224.74), the bonus would have to be \$43 more than \$429,298 (i.e., \$429,341) in order to motivate the CEO to work hard.

4. Suppose it turns out that the CEO values her cost of effort at \$200,000 instead of \$50,000. If this were the case, would the Board's new compensation scheme have its intended effect? If not, what is the minimum level of bonus required in order to incentivize the CEO to work hard?

**SOLUTION:** Plugging \$200,000 into the spreadsheet into cell B5 of the "Salary plus Bonus" worksheet reveals that unless you increase the bonus, the CEO has the incentive to change jobs. Using Solver, you'll find that a bonus of \$701,038 makes the CEO indifferent between working hard and leaving the firm, so the bonus would need to be marginally higher than \$701,038 in order to ensure that the CEO will stay and your firm and decide to work hard (e.g., a bonus of \$701,082 causes expected utility from working hard to be 1,224.75 versus 1,224.74). Note also that the higher cost bonus also improves  $EU_{GoThruMotions}$ , but not by enough to affect behavior; specifically, if cost of effort is \$200,000, then  $EU_{GoThruMotions} = 1,222.50$ .

5. Suppose the CEO values her cost of effort at \$500,000. What is the minimum level of bonus in this case? Should you pay this bonus?

**SOLUTION:** Using Solver, we find that the bonus required in order to incentivize the CEO to work hard is \$1,287,876. We do this by finding a bonus level such that the expected utility of working hard is greater than the expected utility of going through the motions. If the CEO's cost of effort is \$500,000 (as it is in part 5), then the bonus comes out to \$1,287,876. Even though expected profit net of CEO compensation remains higher under the "Work Hard" scenario, it turns out that the CEO will choose instead to go through the motions rather than work hard, since expected utility is 1,280.73 under "Go Through Motions" compared with 1,224.74 under "Work Hard". In this case, the firm would definitely not want to hire a part 5 CEO and instead find a CEO who has a lower cost of effort (e.g.,  $C = \$200,000$  or less as shown in parts 2-4).