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Problem Set #1 Name: SOLUTIONS

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Show your work and write as legibly as possible. Good luck!

1. Determine the first derivative of each of the following functions:

   A. $y = \frac{x}{x-2} \Rightarrow \frac{dy}{dx} = \frac{1}{x-2} - \frac{x}{(x-2)^2} = -\frac{2}{(x-2)^2}$

   B. $y = 2x^3 + \ln x \Rightarrow \frac{dy}{dx} = 6x^2 + \frac{1}{x}$

   C. $y = \frac{1}{2x-1} \Rightarrow \frac{dy}{dx} = -\frac{2}{(2x-1)^2}$

   D. $y = 3 + 10x + 5x^2 \Rightarrow \frac{dy}{dx} = 10 + 10x$

2. Determine the second derivative for each of the four functions listed in problem 1:

   (a) $\frac{dy}{dx} = \frac{1}{x-2} - \frac{x}{(x-2)^2} \Rightarrow \frac{d^2y}{dx^2} = -\frac{2}{(x-2)^2} + \frac{2x}{(x-2)^3} = \frac{4}{(x-2)^3}$

   (b) $\frac{dy}{dx} = 6x^2 + \frac{1}{x} \Rightarrow \frac{d^2y}{dx^2} = 12x - \frac{1}{x^2}$

   (c) $\frac{dy}{dx} = -\frac{2}{(2x-1)^2} \Rightarrow \frac{d^2y}{dx^2} = \frac{8}{(2x-1)^3}$

   (d) $\frac{dy}{dx} = 10 + 10x \Rightarrow \frac{d^2y}{dx^2} = 10$

3. Find the partial derivative of y with respect to x and the partial derivative of y with respect to z in each of the following cases:

   A. $y = 2 + 10z + 3x \Rightarrow \frac{\partial y}{\partial x} = 3$ and $\frac{\partial y}{\partial z} = 10$

   B. $y = 4z^{.5} + 17x^{3.25} \Rightarrow \frac{\partial y}{\partial x} = 55.25x^{2.25}$ and $\frac{\partial y}{\partial z} = 2z^{-.5}$

   C. $y = z^{0.5}x^{0.5} \Rightarrow \frac{\partial y}{\partial x} = .5z^{0.5}x^{-0.5}$ and $\frac{\partial y}{\partial z} = .5z^{-0.5}x^{0.5}$

   D. $y = 4z/(3+x) \Rightarrow \frac{\partial y}{\partial x} = -4z/(3+x)^2$ and $\frac{\partial y}{\partial z} = 4/(3+x)$

4. As the manager of your firm, you wish to determine how many widgets to manufacture, such that profit is maximized. Your chief economist estimates that the fixed costs of operating your manufacturing facility total \$200, whereas variable costs come to $\$5q^2$, where q indicates the total number of widgets produced. The competitively determined price per widget is \$100.

   A. What is total revenue, expressed in terms of q?

      Total revenue is $TR = Pq = \$100q$.

   B. What is total cost, expressed in terms of q?

      Total cost is $TC = \$200 + \$5q^2$.

   C. What is marginal revenue?

      Marginal revenue is $MR = \dfrac{dTR}{dq} = \$100$.

   D. What is marginal cost?

      Marginal cost is $MC = \dfrac{dTC}{dq} = \$10q$.

   E. How many widgets should your company produce; i.e., what value for q *maximizes* total profit? How can you be sure that this is the profit-maximizing, and *not* profit-minimizing value for q?

      The profit maximizing value for q is determined by setting marginal revenue equal to marginal cost; i.e., $\$100 = \$10q \rightarrow q = 10$. Note that this result obtains from maximizing total profit π, where $\pi = TR - TC = \$100q - (200 + 5q^2)$. Thus, $\dfrac{d\pi}{dq} = 100 - 10q = 0 \Rightarrow q = 10$. We know that this is the profit-maximizing value for q because the second order condition is negative; i.e., $\dfrac{d^2\pi}{dq^2} = -10 < 0$.