Binomial Option Pricing Problem Solutions

Finance 4335

Bitcoin, Inc., stock is currently worth \$56. Each year, it can change by a factor of 0.9 or 1.3. The stock pays no dividends, and the annual continuously compounded risk-free interest rate is 4%.

A. Calculate the price of a one-year European put option on Bitcoin, Inc. stock with an exercise price of \$60.

SOLUTION: We can solve this problem by via replicating portfolio and risk neutral valuation approaches.

1. According to the Replicating Portfolio Approach:

$\Delta = \dfrac{P_u - P_d}{uS - dS} = \dfrac{0 - 9.60}{72.80 - 50.40} = -.4286$; and $B = \dfrac{uP_d - dP_u}{e^{r\delta t}(u-d)} = \dfrac{1.3(9.60) - .9(0)}{1.0408(.4)} = 29.98$. Then $V_{RP} = P = \Delta S + B = -.4286(56) + 29.98 = \$5.98$.

2. According to the Risk Neutral Valuation Approach:

The risk neutral probability of an up move is $q = \dfrac{e^{r\delta t} - d}{u - d} = \dfrac{e^{.04} - .9}{1.3 - .9} = .352$. Since the stock is worth $\$56(1.3) = \$72.80$ at the u node and $\$56(.9) = \$50.40$ at the d node, this means that the put is only in the money at the d node; specifically, it is worth \$9.60 at that node. Therefore, the price of a one-year put option is

$$p = e^{-r\delta t}[qp_u + (1-q)p_d] = e^{-.04}[.648(9.60)] = \$5.98.$$

B. Calculate the price of a one-year European call option on Bitcoin, Inc. stock with an exercise price of \$60.

SOLUTION: We can solve this problem by via put-call parity, replicating portfolio, and risk neutral valuation approaches.

1. According to put-call parity,

$$C = P + S - Ke^{-r\delta t} = \$5.98 + \$56 - \$60e^{-.04} = \$4.33.$$

2. According to the Replicating Portfolio Approach:

$\Delta = \dfrac{C_u - C_d}{uS - dS} = \dfrac{12.80 - 0}{72.80 - 50.40} = .5714$ and $B = \dfrac{uC_d - dC_u}{e^{r\delta t}(u-d)} = \dfrac{1.3(0) - .9(12.80)}{1.0408(.4)} = -27.67$. Then $V_{RP} = C = \Delta S + B = .5714(56) - 27.67 = \$4.33$.

3. According to the Risk Neutral Valuation Approach:

The risk neutral probability of an up move is $q = \dfrac{e^{r\delta t} - d}{u - d} = \dfrac{e^{.04} - .9}{1.3 - .9} = .352$. Since the stock is worth $\$56(1.3) = \$72.80$ at the u node and $\$56(.9) = \$50.40$ at the d node, this means that the call is only in the money at the u node; specifically, it is worth \$12.80 at that node. Therefore, the price of a one-year call option is

$$c = e^{-r\delta t}[qc_u + (1-q)c_d] = e^{-.04}[.352(12.80)] = \$4.33.$$

C. Calculate the price of a two-year European put option on Bitcoin, Inc. stock with an exercise price of \$60.

SOLUTION: With two timesteps, there will be 3 terminal nodes – uu, ud, and dd. The share price at these 3 nodes is \$94.64, \$65.52, and \$45.36 respectively. This implies that the put is only in the money at the dd node; specifically, it is worth \$14.64 at that node. Therefore, the price of a two-year put option (based on risk neutral valuation) is

$$p = e^{-2r\delta t}[q^2 p_{uu} + 2q(1-q)p_{ud} + (1-q)^2 p_{dd}] = e^{-.08}(.648^2)(14.64) = \$5.67.$$

D. Calculate the price of a two-year European call option on Bitcoin, Inc. stock with an exercise price of \$60.

SOLUTION: We can solve this problem by applying the put-call parity equation, replicating portfolio approach, or risk neutral valuation. According to put-call parity,

$$c = p + S - Ke^{-2r\delta t} = \$5.67 + \$56 - \$60e^{-.08} = \$6.29.$$

Applying risk neutral valuation, note that the call option is in the money at the uu node (where it is worth \$94.64 – 60 = \$34.64) and at the ud node (where it is worth \$65.52 – 60 = \$5.62); however, it is out of the money at the dd node. Therefore, the price of a two-year call option is

$$c = e^{-2r\delta t}[q^2 c_{uu} + 2q(1-q)c_{ud} + (1-q)^2 c_{dd}] = e^{-.08}[.352^2(34.64) + 2(.352)(.648)(5.62)] = \$6.29.$$