Insurance Economics Class Problem

Finance 4335, February 27, 2024

Suppose that a consumer is subject to the following loss distribution:

State-Contingent Loss (L_s)	Probability of State (p_s)
\$0	1/3
\$2,500	1/3
\$5,000	1/3

This consumer is considering four possible strategies for dealing with this risk. Besides self-insurance, she can also consider the following three insurance policies:

a) Policy A has a \$625 deductible for a premium of \$2,375;
b) Policy B covers 80% of all losses for a premium of \$2,250; and
c) Policy C covers 100% of all losses for a premium of \$3,000.

A. Suppose the consumer's initial wealth is \$10,000, and the only source of risk is the loss distribution. Calculate the expected value of final wealth under the four available risk management strategies (i.e., self-insurance, Policy A, Policy B, and Policy C).

	Self-Insurance	Policy A	Policy B	Policy C
p_s	$W_s = W_0 - L_s$	$W_{A,s} = W_0 - P_A - (L_s - \max(0, L_s - d))$	$W_{B,s} = W_0 - P_B - (1-\alpha)L_s$	$W_{C,s} = W_0 - P_C$
1/3	\$10,000	\$7,625	\$7,750	\$7,000
1/3	\$7,500	\$7,000	\$7,250	\$7,000
1/3	\$5,000	\$7,000	\$6,750	\$7,000
$E(W)$	\$7,500	\$7,208.33	\$7,250	\$7,000

B. What are the premium loadings for Policies A, B, and C?

1. The actuarially fair premium for Policy A is

$$E(I_A) = \sum_{s=1}^{n} p_s(\max(0, L_s - d))$$
$$= (1/3)\$0 + (1/3)\$1,875 + (1/3)\$4,375 = \$2,083.33.$$

Therefore, Policy A's premium loading is $\beta_A = P_A/E(I_A) - 1$ = \$2,375/\$2,083.33 - 1 = 14%.

2. The actuarially fair premium for Policy B is

$$E(I_B) = \sum_{s=1}^{n} p_s \alpha L_s.$$
$$= (1/3)\$0 + (1/3)\$2,000 + (1/3)\$4,000 = \$2,000.$$

Therefore, Policy B's premium loading is $\beta_B = P_B/E(I_B) - 1$ = 2,250/2000 - 1 = 12.5%.

3. The actuarially fair premium for Policy C is

$$E(I_C) = E(L) = \sum_{s=1}^{n} p_s L_s$$
$$= (1/3)\$0 + (1/3)\$2,500 + (1/3)\$5,000 = \$2,500.$$

Therefore, Policy C's premium loading is $\beta_C = P_C/E(I_C) - 1$= 3,000/2,500 - 1 = 20%.

C. Suppose that $U(W) = \ln W$. Which risk management strategy (i.e., self-insurance, Policy A, Policy B, or Policy C) should be selected?

The strategy that yields the highest expected utility of final wealth should be selected. Here are the expected utility calculations for the three strategies:

p_s	Self-Insurance $U(W_s)$	Policy A (Deductible) $U(W_s)$	Policy B (Coinsurance) $U(W_s)$	Policy C $U(W_s)$
33.33%	9.2103	8.9392	8.9554	8.8537
33.33%	8.9227	8.8537	8.8888	8.8537
33.33%	8.5172	8.8537	8.8173	8.8537
$E(U(W))$	8.8834	8.8822	**8.8872**	8.8537

Since Policy B yields the highest expected utility, this policy should be chosen. The utility rankings also indicate that the next best alternative strategy would be self-insurance, then Policy A, and the least preferred alternative would be Policy C.

Keep in mind that the "all else equal" condition required for Arrow's Theorem to hold is violated in this particular numerical example. For starters, note that $E(I_A) > E(I_B)$. Although this is a benefit in an expected utility sense, Policy A also has a higher premium loading (14% versus 12.5%) and therefore costs more per dollar of coverage than Policy B. Policy B has a higher expected utility than Policy A because A's higher premium loading more than offsets the benefit of A's greater coverage.