# Stochastic Dominance Class Problem Solutions

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Consider two risky prospects, $X_l$ and $X_2$, with payoffs given by:

$$X_{1,s} = \begin{cases} \$1 \text{ with probability } 50\% \\ \$9 \text{ with probability } 50\% \end{cases} \text{ and } X_{2,s} = \begin{cases} \$4 \text{ with probability } 99\% \\ \$81 \text{ with probability } 1\% \end{cases}$$

Assume that your initial wealth ($W_0$) is \$0, and your utility $U(W) = \sqrt{W}$.

A. Does one investment first order stochastically dominate the other? Explain why or why not.

In order to establish first order stochastic dominance, it is required that $F(W_{is}) \geq (W_{js})$ for all $s$. In the table below, we check for first order stochastic dominance, and we find that there is no first order stochastic dominance, since when $W_s = 1$, $F(W_{1s}) > F(W_{2s})$, whereas when $W_s = 4$, $F(W_{1s}) < F(W_{2s})$.

| $W_s = W_0 + X_s$ | $f(W_{1s})$ | $F(W_{1s})$ | $f(W_{2s})$ | $F(W_{2s})$ |
|---|---|---|---|---|
| \$1 | 50.00% | 50.00% | 0.00% | 0.00% |
| \$4 | 0.00% | 50.00% | 99.00% | 99.00% |
| \$9 | 50.00% | 100.00% | 0.00% | 99.00% |
| \$81 | 0.00% | 100.00% | 1.00% | 100.00% |

B. Compare these investments once again. Does one investment second order stochastically dominate the other? Explain why or why not.

Risk $j$ second order dominates risk $i$ if $\sum_{s=1}^{n} (F(W_{is}) - F(W_{js})) > 0$. Since $\sum_{s=1}^{n} (F(W_{1s}) - F(W_{2s}))$ $= (.5-0)+(.5-.99)+(1.0-.99)+(1-1) = .02$, it follows that risk 2 second order dominates risk 1.

C. Calculate your expected utility ($E(U(W))$ for both investments.

$E(U(W_1)) = .5(1) + .5(3) = 2$
$E(U(W_2)) = .99(2) + .01(9) = 2.07$

D. Which investment should you choose? Explain why.

You should choose risk 2, since it second order dominates risk 1, which in turn implies that risk 2 also must also have higher expected utility than risk 1. This is true for virtually all arbitrarily risk averse utilities. For example, if $U(W) = \ln W$, then $E(U(W_1)) = .5(2.197) = 1.099$ and $E(U(W_2)) = .99(1.386) + .01(4.394) = 1.416$.