How long does it take to double (triple/quadruple/n-tuple) your money?

by James R. Garven*
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1 Doubling your money (Rule of 72)

Suppose you have $100 and can earn 10% per year on an investment. How long will it take for your initial investment of $100 to be worth $200? To solve this, we need to find the value for t in the following equation:

$$100(1.1)^t = 200. \tag{1}$$

What's challenging about solving equation (1) is that we need to find the value of the unknown exponent t. We could solve for t by trial and error, but that would be unnecessarily cumbersome.

A much easier way to solve for t in equation (1) is to use logarithms. Logarithms were introduced in the early 17th century by a Scottish mathematician named John Napier as a means to simplify calculations like this one. Napier showed that for any two positive real numbers b and x where b is not equal to 1, the logarithm of x to base b, denoted $\log_b(x)$, is the unique real number y such that $b^y = x$. For example, if we divide both sides of equation (1) by 100, this leaves us with the following equation:

$$1.1^t = 2. \tag{2}$$

In equation (2), $b = 1.1$, $y = t$, and x =2. Thus, t is equal to the logarithm of 2 to base 1.1; i.e.,

$$t = \log_{1.1} 2. \tag{3}$$

Spreadsheets and calculators typically include the so-called *common*, or base 10 logarithm function, and the *natural*, or base e logarithm function.[1] Conveniently, equation (3) can be written in terms of the common logarithm:

$$t = \log_{1.1} 2 = \frac{\log_{10} 2}{\log_{10} 1.1} = \frac{0.30103}{.041393} = 7.273. \tag{4}$$

Thus, at a 10% interest rate, a sum doubles in value in 7.273 years. Note that we can validate this result by using 7.273 as the exponent in equation (2):

$$1.1^{7.273} = 2. \tag{5}$$

*James R. Garven is the Frank S. Groner Memorial Chair in Finance and Professor of Finance & Insurance at Baylor University (Address: Foster 320.39, One Bear Place #98004, Waco, TX 76798, telephone: +1 (254) 307-1317, e-mail: James_Garven@baylor.edu).

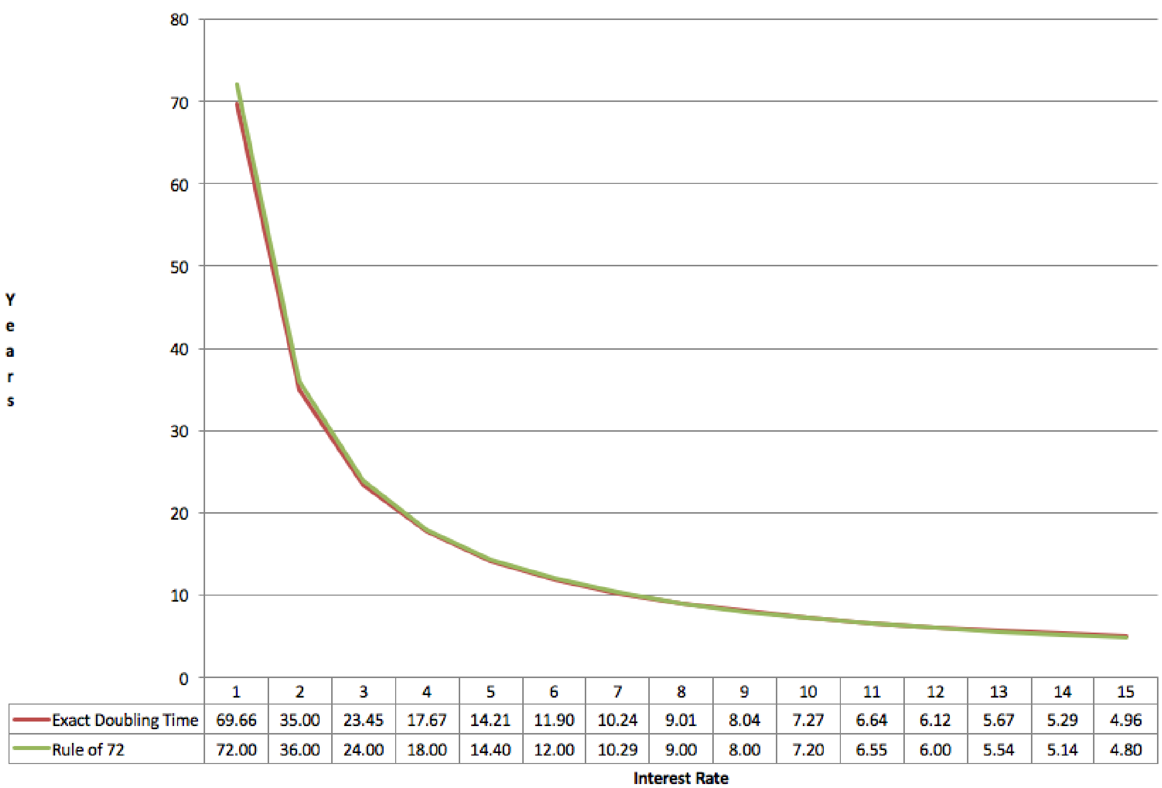
[1] Unfortunately, popular financial calculators such as the TI BA II Plus do not feature a log base 10 function, although they do feature the log base e (natural logarithm) function. However, the log base 10 function is a common feature for scientific calculators.

The so-called "Rule of 72" is a rule of thumb loosely based upon the math shown above. The definition of the Rule of 72 provided by Investopedia is as follows:

> "The 'Rule of 72' is a simplified way to determine how long an investment will take to double, given a fixed annual rate of interest. By dividing 72 by the annual rate of return, investors can get a rough estimate of how many years it will take for the initial investment to duplicate itself."

To see the connection with the math, note that if one multiplies the doubling time found in equation (4) by the interest rate (expressed as a whole number rather than decimal), this product is equal to $r \times t = 10 \times 7.273 = 72.73$; thus $t \cong 72/r = 72/10 = 7.2$. Similarly, if the rate of interest is 8%, then $t = \dfrac{\log_{10} 2}{\log_{10} 1.08} = \dfrac{0.30103}{0.033424} = 9.006 \cong 72/8$, and so forth. Figure 1 shows how robust the Rule of 72 is for annual interest rates ranging from 1% to 15%.

Figure 1: Exact Doubling Time (Red) vs. Rule of 72 (Green)



	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Exact Doubling Time	69.66	35.00	23.45	17.67	14.21	11.90	10.24	9.01	8.04	7.27	6.64	6.12	5.67	5.29	4.96
Rule of 72	72.00	36.00	24.00	18.00	14.40	12.00	10.29	9.00	8.00	7.20	6.55	6.00	5.54	5.14	4.80

2 Tripling, quadrupling, . . . , n-tupling your money

Similarly, we can determine how long it takes for a sum to triple, quadruple, . . . , n–tuple by replacing the number 2 as it appears in equation (3) with the numbers 3, 4, . . . , n. Table 1 indicates how long it takes for a sum to triple, quadruple, quintuple, and dectuple:

Interest Rate	Time to Triple (3x)	Time to Quadruple (4x)	Time to Quintuple (5x)	Time to Dectuple (10x)
1%	110.41	139.32	161.75	231.41
2%	55.48	70.01	81.27	116.28
3%	37.17	46.90	54.45	77.90
4%	28.01	35.35	41.04	58.71
5%	22.52	28.41	32.99	47.19
6%	18.85	23.79	27.62	39.52
7%	16.24	20.49	23.79	34.03
8%	14.27	18.01	20.91	29.92
9%	12.75	16.09	18.68	26.72
10%	11.53	14.55	16.89	24.16
11%	10.53	13.28	15.42	22.06
12%	9.69	12.23	14.20	20.32
13%	8.99	11.34	13.17	18.84
14%	8.38	10.58	12.28	17.57
15%	7.86	9.92	11.52	16.48

Table 1: Number of years required for a sum to triple, quadruple, quintuple, and dectuple at various interest rates.