Pricing Credit Risk using the BSM model Solutions
Finance 4335 Class Problem

Problem Setup: Suppose two banks exist which are identical in all respects except for degree of financial leverage. At date $t = 0$, Bank 1 issues zero coupon deposits with a face value of $B_1 = \$500,000$, whereas bank 2 has issued zero coupon deposits with a face value of $B_2 = \$800,000$. Current asset value in both cases is $V(F) = \$1,000,000$, asset risk for both banks is $\sigma = .4$, and the annual rate of interest is 3%.

One year from today (at date $t = 1$), depositors expect these banks to pay back the face value of deposits with profits earned from investments. However, since bank assets are risky and both banks are limited liability corporations, there is a risk that they won't be paid in full.

Answer the following questions:

1. Suppose there is no deposit insurance. What are the fair market values for the deposits held by Bank 1 and Bank 2 if there is no deposit insurance?

   Solution: In the absence of deposit insurance, depositors are at risk if default occurs; i.e., if $F < B$ at $t = 1$. Consequently, the fair market value of risky deposits is equal to the fair market value of safe deposits minus the value of the limited liability put option; i.e., $V(D) = Be^{-rT} - V(Max[0, B - F])$, where B corresponds to the promised payment and F corresponds to the $t = 1$ value of bank assets.

   We begin by calculating the fair market value for Bank 1 deposits. The value of the limited liability put is $V(Max[0, B - F]) = Be^{-rT}N(-d_2) - V(F)N(-d_1)$,
   $d_1 = \dfrac{\ln(V(F)/B) + (r + .5\sigma^2)T}{\sigma\sqrt{T}} = \dfrac{\ln(1,000,000/500,000) + (.03 + .5(.4^2))1}{.4\sqrt{1}} = 2.0079$
   and $d_2 = d_1 - \sigma\sqrt{T} = 2.0079 - .4\sqrt{1} = 1.6079$. Since $N(-d_1) = N(-2.0079) = .0223$ and $N(-d_2) = N(-1.6079) = .0539$, the value of Bank 1's limited liability put option is $V(Max[0, B - F]) = 500,000e^{-.03}(.0539) - 1,000,000(.0223) = \$3,840.40$, and the fair market value of Bank 1's deposits is $V(D) = Be^{-rT} - V(Max[0, B - F]) = 500,000e^{-.03} - \$3,840.40 = \$481,382.37$.

   Next, we perform the same calculations for Bank 2. Bank 2's
   $d_1 = \dfrac{\ln(1,000,000/800,000) + (.03 + .5(.4^2))1}{.4\sqrt{1}} = .8329$ and $d_2 = d_1 - \sigma\sqrt{T} = .8329 - .4\sqrt{1} = .4329$. Since $N(-d_1) = N(-.08329) = .2025$ and $N(-d_2) = N(-.4329) = .3326$, the value of Bank 2's limited liability put option is
   $V(Max[0, B - F]) = 800,000e^{-.03}(.3326) - 1,000,000(.2025) = \$55,721.88$, and the fair market value of Bank 2's deposits is $V(D) = Be^{-rT} - V(Max[0, B - F]) = 800,000e^{-.03} - \$55,721.88 = \$720,634.55$.

2. What are the values of the limited liability put options held by Bank 1 and Bank 2?

   Solution: As calculated in question 1, the values of the limited liability put options held by Bank 1 and Bank 2 are \$3,840.40 and \$55,721.88 respectively.

3. What are the (risk neutral) probabilities of default for Bank 1 and Bank 2?

   Solution: As calculated in question (1), the risk neutral probabilities of default are $N(-d_2) = N(-1.6079) = .0539$ for Bank 1 and $N(-d_2) = N(-.4329) = .3326$ for Bank 2. These risk neutral default probabilities exceed actual, or "true" default probabilities if expected returns on bank assets exceed the riskless rate of interest. For example, suppose the expected return on these banks' assets is 15%. Then the actual ("true") default probability for Bank 1 is 2.82%, and for Bank 2 it is 23.18%.

4. Calculate yields to maturity and credit risk premiums for Bank 1 and Bank 2.

   Solution: Since $B = V(D)e^{YTM(T)}$, it follows that $YTM = \dfrac{\ln(B/V(D)}{T} = \dfrac{\ln(500,000/481,328.37)}{1} = 3.79\%$ for Bank 1, and $YTM = \dfrac{\ln(800,000/720,634.55)}{1} = 10.45\%$ for Bank 2. The credit risk premium for Bank 1 is $YTM_1 - r = 3.79\% - 3\% = .79\%$ and it is $YTM_2 - r = 10.45\% - 3\% = 7.45\%$ for Bank 2.

5. Suppose the government institutes a risk-based deposit insurance scheme in which bank deposits are fully insured against the risk of default. What are the fair premiums for deposit insurance paid by Bank 1 and Bank 2?

   Solution: Note that the values of the shortfalls for Banks 1 and 2 are represented by the values of these banks' limited liability put options. Since Bank 1 imposes a much smaller risk due to having a much lower degree of financial leverage than Bank 2, Bank 1 is obligated to pay a deposit insurance premium of \$3,840.40, whereas Bank 2 is obligated to pay \$55,721.88.

6. What effect will deposit insurance have on the yields to maturity and credit risk premiums that depositors expect from Bank 1 and Bank 2?

   Solution: Since depositors for both banks no longer have to bear any credit risk, credit premiums go to zero in both cases and yields fall to the riskless rate of interest which is $r = 3\%$.

7. Now suppose the government charges premiums based on the average of the fair premiums that Bank 1 and Bank 2 should pay. Analyze the behavioral effects of such a pricing scheme. Specifically, who wins and who loses, and what incentives are conveyed by such a scheme?

Solution: Since fair premiums are \$3,840.40 for Bank 1 and \$55,721.88 for Bank 2, the average premium is (\$3,840.40 + \$55,721.88)/2 = \$29,781.14. Assuming both banks participate in the risk pool, by charging both banks \$29,781.14, Bank 1 is overpaying by the amount of \$29,781.14 - \$3,840.40 = \$25,940.74, whereas Bank 2 is underpaying by the same amount. Thus, such a pricing scheme forces owners of safe banks to cross-subsidize owners of risky banks. Bank 2 has no incentive to become less risky so long as Bank 1 remains in the risk pool and provides financial compensation for Bank 2's risk taking. Bank 1 has strong incentives to either exit the risk pool (so that it can stop subsidizing Bank 2 risk taking), or become riskier itself.