Finance 4335 Final Exam Formula Sheet

1. **Expected Utility**

$$E\left(U(W)\right) = \sum_{s=1}^{n} p_s U(W_s),$$

where $U(W_s)$ = utility of state-contingent wealth, and p_s = probability of state s.

2. **Risk Premium**

$$\lambda = .5\sigma_W^2 R_A(E(W)).$$

where λ = the dollar value of the risk premium, $E(W)$ = expected value of wealth, σ_W^2 = variance of wealth, and $R_A(E(W))$ = the Arrow-Pratt Absolute Risk Aversion coefficient, evaluated at $E(W)$.

3. **Mean-Variance Theory**

Mean-variance theory may be used to establish that $E(U(X_i)) > E(U(X_j))$ for risk averse utility functions *if and only if* 1) variance is a "complete" risk measure, and 2) one of the following conditions holds:

- $E(X_i) > E(X_j)$ and $\sigma_i^2 < \sigma_j^2$;
- $E(X_i) > E(X_j)$ and $\sigma_i^2 = \sigma_j^2$; or
- $E(X_i) = E(X_j)$ and $\sigma_i^2 < \sigma_j^2$.

4. **Stochastic Dominance**

If X_i stochastically dominates X_j, then $E(U(X_i)) > E(U(X_j))$ for all risk averse utility functions. Here are the formal definitions for first and second order stochastic dominance:

- First Order Stochastic Dominance: Investment i First Order Stochastic Dominates (FOSD) investment j if $F(X_{j,s}) \geq F(X_{i,s})$ for all s.
- Second Order Stochastic Dominance: Investment i Second Order Stochastic Dominates (SOSD) investment j if $\sum_{s=1}^{n} (F(X_{js}) - F(X_{is})) > 0$.

5. **Demand for Insurance**

State-Contingent Wealth: $W_s = W_0 - E(I)(1+\lambda) - L_{u,s}$, where

- W_0 = initial wealth;
- $E(I)$ = expected value of the indemnity;
- λ = % premium loading (note: insurance is actuarially fair if $\lambda = 0$);
- $E(I)(1+\lambda)$ = price of insurance, also known as the "insurance premium"; and
- $L_{u,s}$ = the uninsured loss (note: under full coverage, $L_{u,s} = 0$.

6. **Portfolio and Capital Market Theory**

- σ_i = standard deviation of returns on asset i;
- σ_{ij} = covariance between i and j;
- ρ_{ij} = correlation between i and $j = \sigma_{ij}/\sigma_i\sigma_j$;
- w_i = proportion of portfolio p invested in asset i (note: $\sum_{i=1}^{n} w_i = 1$);
- $E(r_p)$ = expected portfolio return = $\sum_{i=1}^{n} w_i E(r_i)$; if $n = 2, E(r_p) = w_1E(r_1) + w_2E(r_2)$;
- σ_p^2 = portfolio variance = $\sum_{i=1}^{n}\sum_{j=1}^{n} w_i w_j \sigma_{ij}$; when $n = 2$, $\sigma_p^2 = w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1w_2\sigma_{ij}$;
- r_f = the expected rate of return on a risk-free asset;
- $E(r_m)$ = the expected rate of return on the market portfolio;
- σ_m = the standard deviation of return on the market portfolio;
- Capital Market Line: $E(r_p) = r_f + \left[\frac{E(r_m) - r_f}{\sigma_m}\right]\sigma_p$ for mean-variance efficient portfolios;
- $\beta_i = \sigma_{im}/\sigma_m^2$;
- Capital Asset Pricing Model: $E(r_i) = r_f + [E(r_m) - r_f]\,\beta_i$ for individual securities; and
- Sharpe Ratio for Risky Asset j: $\frac{E(r_j) - r_f}{\sigma_j}$.
- Optimal exposure to risky asset j: $\alpha = \frac{(E(r_j) - r_f)}{\sigma_j}\frac{\tau}{\sigma_j} = \text{SharpeRatio}_j\left(\frac{\tau}{\sigma_j}\right)$

7. **Financial Derivatives Pricing Theory**

- "Arbitrage-free" forward price: $K = Se^{rT}$, where S = current price of underlying asset, r = riskless rate of interest, and T = time to maturity
- Black-Scholes-Merton pricing formula for a call option: $C = SN(d_1) - e^{-rT}KN(d_2)$, where K = exercise price, σ = volatility of underlying asset's rate of return, $d_1 = \frac{\ln(S/K) + (r + .5\sigma^2)T}{\sigma\sqrt{T}}$, $d_2 = d_1 - \sigma\sqrt{T}$, $N(d_1)$ = standard normal distribution evaluated at d_1, and $N(d_2)$ = standard normal distribution evaluated at d_2.
- Black-Scholes-Merton pricing formula for a put option: $P = e^{-rT}KN(-d_2) - SN(-d_1)$, and,
- Put-call parity equation: $C + Ke^{-rT} = P + S$

8. **Credit Risk**

- Value of Risky Debt $V(D)$: $V(D) = Be^{-rT} - V[\max(0, B - F)]$
- Value of Limited Liability Put ($V[\max(0, B - F)]$): Determined by applying the Black-Scholes-Merton pricing formula for the value of a put option with underlying asset value $V(F)$ and exercise price B, where F is the value of the firm's assets at date T and B is the promised payment to the firm's creditors at date T;
- Yield to Maturity (YTM): Since $B = V(D)e^{YTM(T)}$, $YTM = \frac{\ln(B/V(D))}{T}$.
- Credit Risk Premium (CRP): $CRP = YTM - r$.