Midterm Exam #1 Formula Sheet

1. **Expected Utility of Wealth ($E(U(W))$)**

   - $E\left(U(W)\right) = \sum_{s=1}^{n} p_s U(W_s)$, where $U(W_s)$ = state-contingent utility of wealth.

2. **Expected Value ($E(X)$), Variance (σ_X^2), Standard Deviation (σ_X), Covariance (σ_{12}), and Correlation (ρ_{12}):**

   - $E(X) = \sum_{s=1}^{n} p_s X_s$, where p_s = the probability of state s and X_s = state s value for X;
   - $\sigma_X^2 = \sum_{s=1}^{n} p_s (X_s - E(X))^2$, and $\sigma_X = \sqrt{\sigma_X^2}$;
   - $\sigma_{12} = \sum_{s=1}^{n} p_s \left(X_{1s} - E(X_1)\right)\left(X_{2s} - E(X_2)\right)$, and $\rho_{12} = \sigma_{12}/\sigma_1\sigma_2$.

3. **Mean-Variance Theory**

   Mean-variance theory may be used to establish that $E(U(X_i)) > E(U(X_j))$ for arbitrarily risk averse utility functions *if and only if* 1) variance is a "complete" risk measure, and 2) one of the following conditions holds:

   - $E(X_i) > E(X_j)$ and $\sigma_i^2 < \sigma_j^2$;
   - $E(X_i) > E(X_j)$ and $\sigma_i^2 = \sigma_j^2$; or
   - $E(X_i) = E(X_j)$ and $\sigma_i^2 < \sigma_j^2$.

4. **Stochastic Dominance Theory**

   If risk X_i stochastically dominates risk X_j, then $E(U(X_i)) > E(U(X_j))$ for arbitrarily risk averse utility functions. Formal definitions for first and second order stochastic dominance are:

   - First Order Stochastic Dominance: Investment i First Order Stochastic Dominates (FOSD) investment j if $F(X_{j,s}) \geq F(X_{i,s})$ for all s.
   - Second Order Stochastic Dominance: Investment i Second Order Stochastic Dominates(SOSD) investment j if $\sum_{s=1}^{n} \left(F(X_{js}) - F(X_{is})\right) > 0$.

5. **Two-asset portfolios: Expected Return, Standard Deviation, and Minimum Risk Portfolio Weighting Scheme**

   - Expected Portfolio Return: $E(r_p) = w_1 E(r_1) + w_2 E(r_2)$.
   - Portfolio Standard Deviation: $\sigma_p = \sqrt{w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1w_2\sigma_{12}}$, where w_1 and $w_2 = 1 - w_1$ correspond to the proportions of wealth invested in assets 1 and 2.
   - Minimum Risk Portfolio Weighting Scheme: $w_1 = \dfrac{\sigma_2^2 - \sigma_{12}}{\sigma_1^2 + \sigma_2^2 - 2\sigma_{12}}$.