The Demand for Insurance

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ABSTRACT: This teaching note provides a simple model of the demand for insurance. We derive the insurance demand equation for a risk-averse individual with logarithmic utility. Comparative statics are obtained, and simple proofs of the Bernoulli principle are shown.

1 Introduction

This teaching note provides a simple, "single risk" model of the demand for insurance.[1] We derive the insurance demand equation for a risk-averse individual with logarithmic utility. Comparative statics are obtained, and simple proofs of the Bernoulli principle[2] are shown.

The note is organized as follows. In the next section, we outline the insurance decision for an arbitrarily risk-averse individual and prove the Bernoulli principle. In the third section, we derive an insurance demand equation based upon the logarithmic utility function. In the fourth section of this note, comparative statics are obtained based upon this utility function. Finally, the appendix provides some numerical examples of the comparative statics.

2 Expected Utility Model of the Demand for Insurance

An individual has initial wealth W_0 and will suffer a loss L with probability π. Thus she owns the lottery $(\langle W_0 - L, W_0\rangle, \langle \pi, 1-\pi\rangle)$. She can take out insurance, in which case she must pay a premium $P = pC$, where p is the premium rate and C is the level of coverage. Thus this individual may exchange the lottery she owns for the lottery

$$(\langle W_0 - pC - L + C, W_0 - pC\rangle, \langle \pi, 1-\pi\rangle). \tag{1}$$

Consider a special case of this lottery, where she full insures risk; i.e., where $C = L$. With full insurance, state contingent wealth is $W_0 - pC$ regardless of whether a state contingent loss occurs; thus she exchanges an uncertain loss (L) for a certain loss (pC).

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[1] By "single risk", we mean that the insurance decision can be made without considering the possibility of other exogenous or endogenous "background" risks. Exogenous background risks include risks to wealth that are outside the individual's control; e.g., the effect of globalization on financial market returns. Endogenous background risks include risks due to informational asymmetries; e.g., moral hazard and adverse selection. Schlesinger (2000) provides an excellent survey of the multiple- as well as single-risk literature on the demand for insurance.

[2] The Bernoulli principle states that a risk-averse individual will fully insure when the premium is actuarially fair; that is, when it has no loading and is therefore equal to the expected value of loss.

We assume that this individual has a von Neumann-Morgenstern utility function $U(W)$. Thus, $U(W)$ is continuous and twice differentiable; i.e., marginal utility is positive and decreasing in wealth. Given these assumptions, insurance will be purchased if and only if a C exists such that the expected utility of being insured is higher than the expected utility of being uninsured; i.e.,

$$\pi U\left(W_0 - pC - L + C\right) + (1-\pi)U\left(W_0 - pC\right) > \pi U\left(W_0 - L\right) + (1-\pi)U\left(W_0\right). \tag{2}$$

Next, we analyze the optimal insurance decision by determining the level of coverage C which maximizes expected utility:

$$\max_C E(U(W)) = \pi U(W_0 - pC - L + C) + (1-\pi)U(W_0 - pC). \tag{3}$$

In order to maximize expected utility, we must solve the first order condition:

$$\pi(1-p)U'(W_0 - pC - L + C) = p(1-\pi)U'(W_0 - pC). \tag{4}$$

We are now ready to prove the Bernoulli principle, which states that a risk-averse individual will fully insure if the insurance premium is actuarially fair.

Bernoulli Principle. Suppose insurance is actuarially fair; i.e., $p = \pi$. Substituting π in place of p in equation (4) and simplifying yields equation (5):

$$U'(W_0 - pC - L + C) = U'(W_0 - pC) \Rightarrow W_0 - pC - L + C = W_0 - pC \tag{5}$$

In order for equation (5) to obtain, it must be the case that $C = L$. Therefore, if the insurance premium is actuarially fair, then full coverage is optimal.

3 Insurance Demand Equation

Next, we derive the insurance demand equation for the case of logarithmic utility.[3] The first order condition given in equation (4) implies that

$$\frac{\pi(1-p)}{W_0 - L + (1-p)C} = \frac{p(1-\pi)}{W_0 - pC}. \tag{6}$$

Solving equation (6) for C,[4] we find that

$$C = \frac{(\pi - 1)pL + (p-\pi)W_0}{p(p-1)}. \tag{7}$$

[3]The logarithmic utility function was selected because of its analytic tractability.

[4]Note that if $p = \pi$, then equation (7) simplifies to $C = L$; i.e., a person with logarithmic utility will fully insure if the insurance premium is actuarially fair.

4 Comparative Statics

4.1 Effect of Changes in Initial Wealth

An interesting question relates to the effect of changes in initial wealth on the demand for insurance. This relationship can be analyzed by differentiating the optimal value for C given by equation (7) with respect to W_0, resulting in the following equation:

$$\frac{\partial C}{\partial W_0} = \frac{p - \pi}{p(p-1)}. \tag{8}$$

If insurance is actuarially fair; i.e., $p{=}\pi$, then changes in initial wealth do not affect insurance demand, since full coverage ($C{=}L$) is optimal, irrespective of the value for W_0! If insurance is unfair; i.e., $p{>}\pi$, then the demand for insurance is inversely related to the level of initial wealth.

4.2 Effect of Changes in the Probability of Loss

Next, we test the relationship between the optimal level of insurance coverage and the probability of loss by differentiating C with respect to π:

$$\frac{\partial C}{\partial \pi} = \frac{Lp - W_0}{p(p-1)}. \tag{9}$$

Since one cannot spend more than initial wealth on insurance, the numerator of this ratio must be negative. We have already determined that the denominator is negative. This implies that the demand for insurance is higher, the higher the probability of loss. Consequently, there is a positive relationship between the optimal level of insurance coverage and the probability of loss.

4.3 Effect of Changes in Loss Severity

What happens to the optimal level of insurance coverage if the severity of a loss changes? We find the answer to this question by differentiating C with respect to L, resulting in the following equation:

$$\frac{\partial C}{\partial L} = \frac{\pi - 1}{p - 1}. \tag{10}$$

We have previously shown that both the numerator and denominator of this ratio are negative. Consequently, the demand for insurance is positively related to loss severity.

4.4 Effect of Changes in the Price of Insurance

Finally, we test the relationship between the optimal level of insurance coverage and the insurance premium by differentiating C with respect to p:

$$\frac{\partial C}{\partial p} = -\frac{Lp^2(\pi - 1) + (p^2 + \pi - 2p\pi)W_0}{(p-1)^2 p^2} = -\frac{\pi(Lp^2 + W_0) + p^2(W_0 - L) - 2p\pi W_0}{(p-1)^2 p^2}. \tag{11}$$

A priori, we expect that the demand for insurance will be inversely related to the price of insurance. In equation (12), the denominator is unambiguously positive. In the numerator, the first term ($\pi(Lp^2+W_0)$) is unambiguously positive, the second term $p^2(W_0-L) \geq 0$ (with the equality holding only when the entire initial wealth $W_0$is at risk), and the third term ($-2p\pi W_0$) is unambiguously negative. Therefore, in order for the demand for insurance to be *positively* related to the price, this may only occur when $2p\pi W_0 > \pi(Lp^2 + W_0) + p^2(W_0 - L)$.

Since it is somewhat difficult to sign equation (12),[5] we will resort to a simple numerical example. Suppose W_0=100, $L = 50$, $\pi = .5$, and let p vary. The following table shows what happens to C (and the coinsurance rate C/L) as p changes:

p	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90
C	472.22	218.75	130.95	83.33	50.00	20.83	-11.90	-62.50	-194.44
C/L	9.44	4.38	2.62	1.67	1.00	0.42	-0.24	-1.25	-3.89

In the above table, note that when insurance is actuarially fair (i.e., if $p = \pi = .5$), then the coinsurance rate $C/L = 1.0$. In other words, full insurance is optimal. However, once the price becomes unfair, then the coinsurance rate falls. In fact, as insurance becomes relatively more expensive, then at some point the consumer will be interested in becoming a net "short seller" of insurance policies. On the other hand, if insurance is offered for less than the actuarially fair price (i.e., if $p < \pi$), then the consumer will prefer to overinsure.

5 References

Hoy, Michael and Arthur J. Robson, 1981, "Insurance as a Giffen Good," *Economics Letters*, Vol. 8, No. 1, 47-51.

Schlesinger, Harris, 2000, "The Theory of Insurance Demand," in Georges Dionne, editor, *Handbook of Insurance* (Boston: Kluwer Academic Publishers).

[5] It is somewhat difficult, but not impossible to sign this equation. Hoy and Robson (1981) have shown that the demand for insurance is *inversely* related to the price if the coefficient of relative risk aversion is less than or equal to one. Since the coefficient of relative risk aversion for the logarithmic utility function is equal to one, this implies that the sign of equation (11) must be negative; i.e., the demand for insurance is inversely related to the insurance premium.

Appendix A

Numerical Comparative Statics of Insurance Demand (based upon equation (7))

Appendix A provides a simple numerical example of the comparative statics of the demand for insurance, based upon equation (7) in the paper. Suppose $W_0 = \$100$, $L = \$50$, $\pi = .50$, and $p = .50$. Inputting these parameters into equation (7) yields

$$C = \frac{(.5-1).5(50)}{.5(.5-1)} = \$50. \tag{A1}$$

Of course, this result (full coverage) is to be expected, since the Bernoulli principle implies that whenever $p = \pi$, then $C = L$.

Suppose W_0, L and π do not change, but the insurer decides to charge a 20% premium loading; i.e., now, $p = .60$. Then

$$C = \frac{(.5-1).6(50) + .1(100)}{.6(.6-1)} = \$20.83. \tag{A2}$$

Here, the level of coverage falls because insurance has become more expensive. The individual is only willing to purchase partial coverage because the increase in premium limits the utility gain obtained from the purchase of an insurance policy.

Suppose L and π do not change, but W_0 increases from \$100 to \$120 and the premium rate is maintained at $p = .60$. Then

$$C = \frac{(.5-1).6(50) + .1(120)}{.6(.6-1)} = \$12.50. \tag{A3}$$

The level of coverage falls with an increase in initial wealth because of diminishing marginal utility. Basically, the utility loss from limiting insurance coverage is less severe for a "wealthy" person than it is for an otherwise identical "poor" person.

Suppose $W_0 = \$120$, $\pi = .50$, $p = .60$, and L increases from \$50 to \$60. Then

$$C = \frac{(.5-1).6(60) + .1(120)}{.6(.6-1)} = \$25. \tag{A4}$$

The level of coverage increases because severity has increased; the larger loss implies that the individual gains more utility from transferring risk to an insurer, even though the premium is not actuarially fair.

Finally, suppose $W_0 = \$120$, $L = \$60$, $p = .60$, and π increases from .50 to .60. Then

$$C = \frac{(.6-1).6(60) + .1(120)}{.6(.6-1)} = \$60. \tag{A5}$$

The level of coverage increases because there is a higher risk of loss. Also, we know from the Bernoulli principle since $p = \pi$, full coverage ($C = L$) is optimal.