## Dynamic Adverse Selection Class Problem (Solutions)

The Government Employee Car Krash Organization (also known as "GECKO") does business in Estonia, where automobile liability insurance is not compulsory; i.e., licensed drivers are allowed to (as a matter of their own volition) decide whether or not to purchase such insurance. However, the Estonian insurance commissioner requires that GECKO must offer full coverage and charge the same premium to all of its policyholders. Furthermore, the premium must be set such that the dollar value of GECKO's expected profit from selling insurance is equal to $\$ 0$.

GECKO estimates that the accident probabilities for the following five driver types are as follows (for simplicity, assume that there is only one of each driver type):

| Driver Type | Probability of Accident |
| :---: | :---: |
| Cautious Caroline | $5 \%$ |
| Nervous Nora | $25 \%$ |
| Average Arvis | $30 \%$ |
| Aggressive Anna | $35 \%$ |
| Hot Rod Henriks | $40 \%$ |

The dollar value of initial wealth and loss due to an accident for all driver types are $\$ 100,000$ and $\$ 40,000$, respectively. This implies that if an accident occurs, then the dollar value of uninsured wealth falls to $\$ 60,000$. Furthermore, utility $U(W)=\sqrt{W}$ for all driver types. All drivers can pay the same insurance premium $(P)$, which will fully cover accident-related loss.

1. Suppose that GECKO initially sets the premium at $P=\$ 10,800$. This premium will enable GECKO to comply with Estonian insurance regulations so long as all five driver types purchase insurance. Calculate the cross-subsidies that are implied by such a pricing scheme if all five purchase coverage.

| Driver Type | Probability of Accident | Expected Loss | Subsidy | EU(uninsured) | EU(insured) |  |
| :--- | :--- | ---: | ---: | ---: | ---: | ---: |
| Cautious Caroline | $5 \%$ | $\$ 2,000$ | $-\$ 8,800$ | 312.66 | 298.66 |  |
| Nervous Nora | $25 \%$ | $\$ 10,000$ | $-\$ 800$ | 298.41 | 298.66 |  |
| Average Arvis | $30 \%$ | $\$ 12,000$ | $\$ 1,200$ | 294.84 | 298.66 |  |
| Aggressive Anna | $35 \%$ | $\$ 14,000$ | $\$ 3,200$ | 291.28 | 298.66 |  |
| Hot Rod Henriks |  | $40 \%$ | $\$ 16,000$ | $\$ 5,200$ | 287.72 | 298.66 |
|  |  | $\$ 54,000$ |  |  |  |  |
|  | Total Expected Loss |  | $\$ 10,800$ |  |  |  |
|  | Combined Premium |  |  |  |  |  |

Figure 1: Cross-Subsidies when all 5 driver types purchase coverage for $\$ 10,800$
2. The situation described in part 1 of this problem is not a stable equilibrium since Cautious Caroline has higher expected utility if she opts out of purchasing coverage for a price of $\$ 10,800$. Since the expected loss costs for the remaining four clients now totals $\$ 52,000$, the new combined premium must therefore increase from $\$ 10,800$ to $\$ 52,000 / 4=\$ 13,000$. Calculate the cross-subsidies that are implied by a $\$ 13,000$ price if the four remaining clients purchase coverage.

| Driver Type | Probability of Accident | Expected Loss | Subsidy | EU(uninsured) | EU(insured) |
| :--- | ---: | ---: | ---: | ---: | ---: |
| Nervous Nora | $25 \%$ | $\$ 10,000$ | $-\$ 3,000$ | 298.41 | 294.96 |
| Average Arvis | $30 \%$ | $\$ 12,000$ | $-\$ 1,000$ | 294.84 | 294.96 |
| Aggressive Anna | $35 \%$ | $\$ 14,000$ | $\$ 1,000$ | 291.28 | 294.96 |
| Hot Rod Henriks |  | $40 \%$ | $\$ 16,000$ | $\$ 3,000$ | 287.72 |
|  |  | $\$ 52,000$ |  |  |  |

Figure 2: Cross-Subsidies when the 4 remaining driver types purchase coverage for $\$ 13,000$
3. The situation described in part 2 of this problem is also not a stable equilibrium since Nervous Nora has higher expected utility if she opts out of purchasing coverage for a price of $\$ 13,000$. Since the expected loss costs for the remaining three clients total $\$ 42,000$, the new combined premium must, therefore, increase from $\$ 13,000$ to $\$ 42,000 / 3=\$ 14,000$. Calculate the cross-subsidies that are implied by a $\$ 14,000$ price if the three remaining clients purchase coverage.

| Driver Type | Probability of Accident | Expected Loss | Subsidy | EU(uninsured) | EU(insured) |
| :--- | ---: | ---: | ---: | ---: | ---: |
| Average Arvis | $30 \%$ | $\$ 12,000$ | $-\$ 2,000$ | 294.84 | 293.26 |
| Aggressive Anna | $35 \%$ | $\$ 14,000$ | $\$ 0$ | 291.28 | 293.26 |
| Hot Rod Henriks |  | $40 \%$ | $\$ 16,000$ | $\$ 2,000$ | 287.72 |
|  |  |  |  |  |  |
|  | Total Expected Loss |  | $\$ 2,000$ |  |  |
|  | Combined Premium |  | $\$ 14,000$ |  |  |

Figure 3: Cross-Subsidies when the 3 remaining driver types purchase coverage for $\$ 15,000$
4. The situation described in part 3 of this problem is also not a stable equilibrium since Average Arvis has higher expected utility if he opts out of purchasing coverage for a price of $\$ 14,000$. Since the expected loss costs for the remaining two clients total $\$ 30,000$, the new combined premium must, therefore, increase from $\$ 14,000$ to $\$ 30,000 / 2=\$ 15,000$. Calculate the cross-subsidies that are implied by a $\$ 15,000$ price if the two remaining clients purchase coverage.

| Driver Type | Probability of Accident | Expected Loss | Subsidy | EU(uninsured) | EU(insured) |  |
| :--- | :--- | ---: | ---: | ---: | ---: | ---: |
| Aggressive Anna | $35 \%$ | $\$ 14,000$ | $-\$ 1,000$ | 291.28 | 291.55 |  |
| Hot Rod Henriks |  | $40 \%$ | $\$ 16,000$ | $\$ 1,000$ | 287.72 | 291.55 |
|  | Total Expected Loss | $\$ 30,000$ |  |  |  |  |
|  | Combined Premium | $\$ 15,000$ |  |  |  |  |

Figure 4: Cross-Subsidies when the 2 remaining driver types purchase coverage for $\$ 15,000$
The situation depicted by Figure 4 depicts a stable equilibrium in the sense that neither client has an incentive to defect. Even though Aggressive Anna cross-subsidizes Hot Rod Henriks' premium cost by $\$ 1,000$, she still prefers "unfair" coverage over no coverage at all.

