Finance 4335 (Spring 2024) Midterm 1 Synopsis

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I. The foundational principle upon which Finance 4335, Part 1 is based is that decision-makers vary in terms of their risk-bearing preferences. While our primary focus has been on modeling risk averse behavior (where decision-makers dislike risk but are willing to bear risk if properly compensated), we also considered risk neutrality (where decision-makers care only about expected wealth and are indifferent about risk) and risk loving behavior (where decision-makers not only prefer to bear risk but are willing to expend resources for the opportunity to do so).

II. Regardless of whether one is risk averse, risk neutral, or risk loving, the foundation for decision-making under risk and uncertainty is expected utility. Given a choice among various risky alternatives, one selects the choice that yields the highest utility ranking.

A. If one is risk averse, then $E(W) > W_{CE}$ and the difference between $E(W)$ and W_{CE} is equal to the risk premium λ. Here are some practical implications - under risk aversion, insurance buyers may find actuarially unfair insurance premiums financially attractive. If a bettor is risk averse, he or she will not be willing to pay more than the certainty equivalent of wealth for a bet on a sporting event or a game of chance, such as rolling dice or tossing a coin.

B. If one is risk neutral, then $E(W) = W_{CE}$ and $\lambda = 0$; risk is inconsequential, and all that matters is that the expected value of wealth is maximized.

C. If one is risk loving, then $E(W) < W_{CE}$ and $\lambda < 0$; i.e., such a person is willing to pay for the opportunity to (on average) lose money. This is because risk is, by definition, a "*sought-after*" attribute for someone with risk loving preferences.

III. We discussed different methods for calculating λ for risk averse decision-makers.

A. The initial method used requires 1) calculating expected utility $E(U(W)$, 2) setting expected utility equal to the certainty-equivalent of wealth; i.e., $E(U(W)) = U(W_{CE})$, 3) solving the $E(U(W)) = U(W_{CE})$ equation for W_{CE}, and 4) determining λ by calculating the difference between $E(W)$ and W_{CE}. For example, suppose the decision-maker has $U(W) = \sqrt{W}$, $E(W) = \$110$, and $E(U(W)) = 10$. Then $W_{CE} = 100$, and $\lambda = E(W) - W_{CE} = \$110 - \$100 = \$10$.

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B. An alternative method for calculating λ involves evaluating the Arrow-Pratt coefficient ($R_A(W) = -U''/U'$) at the expected value of wealth and multiplying it by half of the variance of wealth. Suppose the variance of wealth $\sigma_W^2 = 4{,}400$. Then for $U(W) = \sqrt{W}$, $R_A(W) = .5/W$ and $\lambda \cong .5\sigma_W^2 R_A(E(W)) = .5(4,400)(.5/\$110) = \$10$.

1. The Arrow-Pratt method provides important intuitive insights into the determinants of risk premiums. We find that risk premiums depend on two factors: 1) the <u>magnitude of the risk itself</u> (as indicated by variance), and 2) the <u>degree</u> to which the decision-maker is <u>risk averse</u> (as indicated by the Arrow-Pratt coefficient). For example, suppose $U(W) = \ln W$. Then $R_A(W) = 1/W$ and $\lambda \cong .5\sigma_W^2 R_A(E(W)) = .5(4,400)(1/\$110) = \$20$. Thus, the logarithmic decision-maker is *twice* as risk averse than the square root decision-maker.

IV. We discussed "special cases" of expected utility; specifically, the <u>mean-variance</u> and the <u>stochastic dominance</u> models. If we impose various restrictive assumptions upon expected utility, then these models emerge as "special cases".

A. As long as the various restrictive assumptions required by these models apply, we can be confident that if risk X "dominates" risk Y, then the expected utility for X is greater than the expected utility for Y; a result which applies to *all arbitrarily risk averse decision-makers*.

B. Of these two models, the mean-variance model is <u>more restrictive</u> than stochastic dominance. Indeed, the mean-variance model is *not* an appropriate method for risk evaluation under a variety of circumstances. For example, if one risk has a *higher mean and variance* than another risk, then we need further information about the decision-maker's utility function in order to determine which risk is preferred; just knowing the mean and variance is *not sufficient* in such a case.

C. The mean-variance model implicitly assumes that *risks are symmetrically distributed and have "thin" tails*; examples of such distributions include the <u>binomial distribution</u> in the discrete setting and the <u>normal distribution</u> in the continuous setting. However, if the underlying distribution is <u>skewed</u> or <u>fat-tailed</u>, then it is *not appropriate* to rank-order risks based on the mean-variance framework because variance only partially captures risk.

1. To illustrate this, we considered a numerical example (see pp. 6-8 of the Decision Making Under Risk and Uncertainty (Part 3) lecture note) in which a positively

skewed risk with a lower mean and a higher variance has higher expected utility than a symmetrically distributed risk with higher mean and lower variance.

D. Stochastic dominance is more "robust" than the mean-variance model because unlike the mean-variance model, the stochastic dominance model properly accounts for broader risk characteristics such as skewness and kurtosis.