# Baylor University
# Hankamer School of Business
# Department of Finance, Insurance & Real Estate

Risk Management, Spring 2024 Name: SOLUTIONS
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Problem Set 1

Show your work and write as legibly as possible. Good luck!

**Problem 1 (12 points):** Find the derivative of the following function with respect to $x$:

$$f(x) = 4x^5 - 3x^4 + 6x^3 - 8x^2 + 2x - 1$$

To find the derivative, we differentiate each term of the function with respect to $x$ using the power rule:

$$f'(x) = \frac{d}{dx}(4x^5) - \frac{d}{dx}(3x^4) + \frac{d}{dx}(6x^3) - \frac{d}{dx}(8x^2) + \frac{d}{dx}(2x) - \frac{d}{dx}(1)$$

Applying the power rule:

$$f'(x) = 20x^4 - 12x^3 + 18x^2 - 16x + 2$$

**Problem 2 (12 points):** Find the derivative of the following function with respect to $x$:

$$h(x) = (5x^2 - 3x + 2)^{\frac{1}{3}}$$

To find the derivative of the given function, we use the chain rule. Let's define $u = 5x^2 - 3x + 2$:

$$h'(x) = \frac{d}{dx}(u^{\frac{1}{3}})$$

Using the chain rule:

$$h'(x) = \frac{1}{3}u^{-\frac{2}{3}}\frac{du}{dx}$$

Next, find the derivative of $u$ with respect to $x$ using the power rule:

$$\frac{du}{dx} = 10x - 3$$

Substitute back into the chain rule expression:

$$h'(x) = \frac{1}{3}(5x^2 - 3x + 2)^{-\frac{2}{3}}(10x - 3)$$

**Problem 3 (12 points):** Find the derivative of the following function with respect to $x$:

$$k(x) = e^{(5x)}$$

To find the derivative of the exponential function, we use the chain rule. Recall that the derivative of $e^u$ with respect to $u$ is $e^u$:

$$k'(x) = \frac{d}{dx}(e^{(5x)})$$

Using the chain rule:

$$k'(x) = e^{(5x)} \frac{d}{dx}(5x)$$

Applying the constant rule:

$$k'(x) = e^{(5x)} \cdot 5$$

Simplifying further:

$$k'(x) = 5e^{(5x)}$$

**Problem 4 (12 points):** Find the derivative of the following function with respect to $x$:

$$f(x) = \ln(3x^2 - 2x + 4)$$

To find the derivative of the natural logarithm function, we use the chain rule. Recall that the derivative of $\ln(u)$ with respect to $u$ is $\frac{1}{u}$:

$$f'(x) = \frac{d}{dx}(\ln(3x^2 - 2x + 4))$$

Using the chain rule:

$$f'(x) = \frac{1}{3x^2 - 2x + 4} \frac{d}{dx}(3x^2 - 2x + 4)$$

Applying the power rule:

$$f'(x) = \frac{1}{3x^2 - 2x + 4}(6x - 2)$$

Simplifying further:

$$f'(x) = \frac{6x - 2}{3x^2 - 2x + 4}$$

**Problem 5 (12 points):** Find the partial derivative of the following function with respect to $x$:

$$f(x, y) = x^2 + 3xy + y^3$$

To find the partial derivative with respect to $x$, treat $y$ as a constant and differentiate the function with respect to $x$:

$$\frac{\partial f}{\partial x} = \frac{d}{dx}(x^2 + 3xy + y^3) = 2x + 3y$$

To find the partial derivative with respect to $y$, treat $x$ as a constant and differentiate the function with respect to $y$:

$$\frac{\partial f}{\partial y} = \frac{d}{dy}(x^2 + 3xy + y^3) = 3x + 3y^2$$

**Problem 6 (40 points):** As the manager of your firm, you wish to determine how many gadgets to manufacture, such that profit is maximized. Your chief economist estimates that the fixed costs of operating your manufacturing facility total \$60,000, whereas variable costs come to $\$6x^2$, where $x$ indicates the total number of gadgets produced. The competitively determined price per gadget is \$1,800.

A. What is total revenue, expressed in terms of $x$?

   Total revenue is $TR = Px = \$1,800x$.

B. What is total cost, expressed in terms of $x$?

   Total cost is $TC = \$60,000 + \$6x^2$.

C. What is marginal revenue?

   Marginal revenue is $MR = \dfrac{dTR}{dx} = \$1,800$.

D. What is marginal cost?

   Marginal cost is $MC = \dfrac{dTC}{dx} = \$12x$.

E. How many gadgets should your company produce; i.e., what value for $x$ maximizes total profit? How can you be sure that this is the profit-maximizing, and not profit-minimizing value for $x$?

   The profit-maximizing value for $x$ is determined by setting marginal revenue equal to marginal cost; i.e., $1,800 = 12x \rightarrow x = 150$. Note that this result obtains from maximizing total profit $\pi$, where $\pi = TR - TC = 1,800x - (60,000 + 6x^2)$. Thus, $\dfrac{d\pi}{dx} = 1,800 - 12x = 0 \rightarrow x = 150$. We know that this is the profit-maximizing value for $x$ because the second order condition is negative; i.e., $\dfrac{d^2\pi}{dx^2} = -12 < 0$.