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Risk Management
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Problem Set 4

Name: SOLUTIONS

Show your work and write as legibly as possible. Good luck!

Miles and John are considering two mutually exclusive risky investments, 1 and 2, with payoffs given by:

$$W_{1,s} = \begin{cases} \$20 \text{ with probability } 20\% \\ \$60 \text{ with probability } 50\% \\ \$100 \text{ with probability } 30\% \end{cases} \quad \text{and } W_{2,s} = \begin{cases} \$40 \text{ with probability } 55\% \\ \$80 \text{ with probability } 45\% \end{cases}$$

Suppose that Miles's initial wealth $W_0 = \$0$, and his utility $U(W) = W^{.5}$.

A. Calculate Miles's expected utility ($E(U(W))$ for both investments.

SOLUTION:

$$E(U(W_{1,s})) = .2\sqrt{20} + .5\sqrt{60} + .3\sqrt{100} = 7.76$$
$$E(U(W_{2,s})) = .55\sqrt{40} + .45\sqrt{80} = 7.49$$

B. Now suppose that John also has initial wealth $W_0 = \$0$, but his utility function is $U(W) = \ln W$. Calculate John's expected utility ($E(U(W))$ for both investments.

SOLUTION:

$$E(U(W_{1,s})) = .2\ln(20) + .5\ln(60) + .3\ln(100) = 4.03$$
$$E(U(W_{2,s})) = .55\ln(40) + .45\ln(80) = 4.00$$

C. Does either investment first order stochastically dominate the other? Explain why or why not.

SOLUTION: We check for first order stochastic dominance by comparing the cumulative probabilities:

W_s	$f(W_{1,s})$	$F(W_{1,s})$	$f(W_{2,s})$	$F(W_{2,s})$
\$20	20%	20%	0%	0%
\$40	0%	20%	55%	55%
\$60	50%	70%	0%	55%
\$80	0%	70%	45%	100%
\$100	30%	100%	0%	100%

Since $F(W_{1,s})$ initially exceeds $F(W_{2,s})$ and is subsequently less than $F(W_{2,s})$, there is no first order stochastic dominance.

D. Compare these investments once again. Is there second order stochastic dominance? Explain why or why not.

SOLUTION: We check for second order stochastic dominance by adding an additional column to the table shown in part A where we calculate $\sum_{s=1}^{n} (F(W_{2,s}) - F(W_{1,s}))$:

W_s	$f(W_{1,s})$	$F(W_{1,s})$	$f(W_{2,s})$	$F(W_{2,s})$	$F(W_{2,s}) - F(W_{1,s})$
\$20	20%	20%	0%	0%	-20%
\$40	0%	20%	55%	55%	35%
\$60	50%	70%	0%	55%	-15%
\$80	0%	70%	45%	100%	30%
\$100	30%	100%	0%	100%	0%

$$\sum_{s=1}^{n} (F(W_{2,s}) - F(W_{1,s})) = 30\%$$

Thus, we find that investment 1 second order stochastically dominates investment 2.

E. Which investment should Miles choose? Explain why.

SOLUTION: Since investment 1 stochastically dominates investment 2, this ensures that Miles will have higher expected utility from investment 1 than from investment 2 (as shown in Part A of this problem). Therefore, Miles prefers (and will choose) investment 1 instead of investment 2.

F. Which investment should John choose? Explain why.

SOLUTION: Even though John is *more* risk averse than Miles (since as we showed in class, logarithmic utility is more risk averse than square root utility), John prefers (and will choose) investment 1 instead of investment 2 for the very same reason as Miles. Specifically, stochastic dominance ensures that John will have higher expected utility from investment 1 than from investment 2 (as shown in Part B of this problem).