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Risk Management
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Problem Set 5
Show your work and write as legibly as possible. Good luck!
Assume a consumer has initial wealth of $\$ 80,000$, utility of wealth $U(W)=\sqrt{W}$, and her wealth is subject to the following loss distribution:

| Loss Amount $\left(\boldsymbol{L}_{s}\right)$ | Probability $\left(\boldsymbol{p}_{\boldsymbol{s}}\right)$ |
| :--- | :--- |
| $\$ 0$ | $50 \%$ |
| $\$ 10,000$ | $30 \%$ |
| $\$ 60,000$ | $20 \%$ |

A. Determine the expected utility of wealth, assuming the consumer is uninsured.

SOLUTION: In order to determine the expected utility of wealth for this consumer, we must calculate the state contingent value of wealth $\mathrm{W}_{s}=W_{0}-L_{s}$, and then use this information to determine the state contingent value of the utility of wealth $U\left(W_{s}\right)$ :

| $p_{s}$ | $L_{s}$ | $p_{s} L_{s}$ | $W_{s}$ | $p_{s} W_{s}$ | $U\left(W_{s}\right)$ | $p_{s} U\left(W_{s}\right)$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $50 \%$ | $\$ 0$ | $\$ 0$ | $\$ 80,000$ | $\$ 0$ | 282.8427 | 141.4214 |
| $30 \%$ | $\$ 10,000$ | $\$ 3,000$ | $\$ 70,000$ | $\$ 21,000$ | 268.5751 | 79.3725 |
| $20 \%$ | $\$ 60,000$ | $\$ 12,000$ | $\$ 20,000$ | $\$ 12,000$ | 141.4214 | 28.2843 |
|  | $E(L)$ | $\$ 15,000$ | $E(W)$ | $\$ 33,000$ | $E(U(W))$ | 249.0782 |

Therefore, expected utility is 249.0782 .
B. Calculate the actuarially fair price for a full (100\%) coverage insurance policy.

SOLUTION: The actuarially fair price for full coverage insurance is simply the expected value of loss, which we calculated in the above table as $\$ 15,000$.
C. Show that it is optimal for this consumer to purchase a full coverage insurance policy at its actuarially fair price by comparing expected utility in the absence of insurance with expected utility in the presence of insurance.

SOLUTION: To determine whether this consumer would purchase a full coverage policy at the actuarially fair price, we must calculate the expected utility of of such a policy. Since the actuarially fair price for an actually fair full policy is $\$ 15,000$, by purchasing
such a policy, this consumer will have certain wealth of $\$ 65,000$. The utility of $\$ 65,000$ is $U(W)=\sqrt{65,000}=254.9510$. Consequently, this consumer will prefer the full coverage policy to remaining uninsured. What we have provided here is a simple numerical "proof" of the Bernoulli hypothesis for a consumer with square root utility. However, note that the application of any arbitrary risk averse utility (i.e., not just the square root function), would yield exactly the same preference ordering as in the example provided here.
D. If only full coverage insurance policies are available in the market, what is the maximum price that this consumer is willing to pay for such a policy?

SOLUTION: To answer this question, we must calculate the certainty equivalent of wealth. Since the expected utility of remaining uninsured is 249.0782 , this means that the certainty equivalent of wealth is $249.0782^{2}=\$ 62,039.93$. Therefore, the maximum premium that this consumer is willing to pay is $\$ 80,000-\$ 62,039.93=\$ 17,960.07$.
E. Suppose this consumer may choose one of the following four risk management strategies:

1) Policy A fully covers all losses for a price of $\$ 18,000$;
2) Policy B has a $\$ 8,000$ deductible and costs $\$ 13,200$;
3) Policy C covers $80 \%$ of all losses for a price of $\$ 14,400$; and
4) Self-insure.

Which of these four strategies will this consumer choose? Explain why.
SOLUTION: We begin by calculating state-contingent wealth under the four alternatives of self-insurance, Policy A, Policy B, and Policy C:

## State Contingent Wealth

| $\boldsymbol{p}_{\boldsymbol{s}}$ | $\boldsymbol{L}_{s}$ | Self- <br> insure | Policy A | Policy B | Policy C |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $50 \%$ | $\$ 0$ | $\$ 80,000$ | $\$ 62,000$ | $\$ 66,800$ | $\$ 65,600$ |
| $30 \%$ | $\$ 10,000$ | $\$ 70,000$ | $\$ 62,000$ | $\$ 58,800$ | $\$ 63,600$ |
| $20 \%$ | $\$ 60,000$ | $\$ 20,000$ | $\$ 62,000$ | $\$ 58,800$ | $\$ 53,600$ |
| $\mathbf{E}()$. | $\$ \mathbf{1 5 , 0 0 0}$ | $\mathbf{\$ 6 5 , 0 0 0}$ | $\$ \mathbf{6 2 , 0 0 0}$ | $\$ \mathbf{6 2 , 8 0 0}$ | $\$ 62,600$ |

As we can see from the above table, the highest expected wealth is obtained from selfinsuring, which should be apparent because this is the only way to avoid paying an actuarially unfair insurance premium. In order to determine which policy should be purchased, we must calculate the expected utility associated with these four alternatives:

## Expected Utility

| $\boldsymbol{p}$ | $\boldsymbol{L}$ | Self- <br> insure | Policy A | Policy B | Policy C |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $50 \%$ | $\$ 0$ | 282.8427 | 248.9980 | 258.4570 | 256.1250 |
| $30 \%$ | $\$ 10,000$ | 264.5751 | 248.9980 | 242.4871 | 252.1904 |
| $20 \%$ | $\$ 60,000$ | 141.4214 | 248.9980 | 242.4871 | 231.5167 |
| Expected <br> Utility |  | $\mathbf{2 4 9 . 0 7 8 2}$ | $\mathbf{2 4 8 . 9 9 8 0}$ | $\mathbf{2 5 0 . 4 7 2 0}$ | $\mathbf{2 5 0 . 0 2 3 0}$ |

Since Policy B has the highest expected utility, we would expect the consumer to purchase that policy. Note that since the premium loading (i.e., percentage mark-up from actuarially fair value) for all three policies comes to $20 \%$, this result is completely consistent with the prediction of the Arrow Theorem, which indicates that holding a (positive) premium loading constant, partial coverage is always preferred to full coverage, and that a deductible policy will be the preferred policy choice. Although we showed the Arrow theorem for the case of a square root consumer, this principle obtains for arbitrarily risk averse consumers.

