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Risk Management
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Problem Set 6

Name: SOLUTIONS

Show your work and write as legibly as possible. Good luck!

Problem 1

The following table lists the state-contingent returns on Security A ($r_{A,s}$) and Security B ($r_{B,s}$):

State of Economy	p_s	$r_{A,s}$	$r_{B,s}$
Bust	50%	-0.20	+0.25
Boom	50%	+0.40	-0.05

A. What are the expected returns for Security A and Security B?

SOLUTION: $E(r_A) = \sum_{s=1}^{n} p_s r_{A,s}$= .5(-.2) + .5(.4) = .10; and

$E(r_B) = \sum_{s=1}^{n} p_s r_{B,s}$= .5(.25) + .5(-.05) = .10.

B. What are the standard deviations of the returns for Security A and Security B?

SOLUTION:

$\sigma^2_{r_A} = \sum_{s=1}^{n} p_s (r_{A,s} - E(r_\mathrm{A}))^2 = .5(\text{-}.2 - .10)^2 + .5(.4 - .10)^2 = .09$; therefore σ_{r_A}= .30; and

$\sigma^2_{r_B} = \sum_{s=1}^{n} p_s (r_{B,s} - E(r_B))^2 = .5(.25 - .10)^2 + .5(\text{-}.05 - .10)^2 = .02$; therefore σ_{r_B}= .150.

C. Find the expected return and standard deviation for the least possible risky combination of Security A and Security B. What is the composition of this portfolio (i.e., find the security weights w_A and w_B)?

SOLUTION:

$\sigma_{AB} = \sum_{s=1}^{n} p_s(r_{As} - E(r_A))(r_{Bs} - E(r_B))$ = .5(-.2 -.10)(.25 -.10) + .5(.40 -.10)(-.05-.10) = -.045.

Therefore, $w_A = \dfrac{\sigma_B^2 - \sigma_{AB}}{\sigma_A^2 + \sigma_B^2 - 2\sigma_{AB}} = \dfrac{.0225 + .045}{.09 + .0225 + .09}$= 1/3. Consequently, $w_B = 1 - w_A$= 2/3, and r_{mvp} = .10. Furthermore, since ρ_{AB} = -.045/(.30)(.15) = -1, the standard deviation of the least possible risky combination of security A and security B is zero.

D. Suppose your initial wealth is \$1,000 and that you can borrow or lend up to \$1,000 at the riskless rate of interest of 3% during the next year. Given this information, describe the most profitable *riskless* trading strategy which can be implemented, and calculate the profit from implementing this strategy.

SOLUTION: The most obvious riskless investment strategy would involve investing your initial wealth of \$1,000 in a riskless bond that yields 3%. Furthermore, since you can borrow up to \$1,000, you could also lever this strategy by investing \$2,000 at 3% and then paying back the principal plus interest on the \$1,000 loan. However, since the the opportunity cost of capital for a riskless investment *is* the riskless rate of interest, these strategies do not increase your net worth; i.e., their net present values are \$0.

It is possible to increase your net worth without taking any risk by investing your initial wealth of \$1,000 plus an additional \$1,000 of borrowed money in the minimum variance portfolio; the value of such an investment after 1 year is $\$2,000e^{.10} - 1,000e^{.03}$= \$2,210.34 – \$1,072.51 = \$1,137.83, which implies an expected return totaling 13.78%. The net present value of this riskless arbitrage strategy is $NPV = \$1,137.83e^{-.07}$ - \$1,000 = \$60.91.

Problem 2

Suppose you have two stocks in your portfolio, *Maxima* and *Minima*. The expected return of *Maxima* is 12% and the expected return of *Minima* is 6%. The standard deviation of *Maxima* is 20% and the standard deviation of *Minima* is 12%. The correlation between the two securities is zero. Suppose the riskless asset has an expected return of 3%.

A. What is the mean and standard deviation of the minimum variance portfolio combination of *Maxima* and *Minima*?

SOLUTION: The ratio given by $w_{\text{Minima}} = \dfrac{\sigma^2_{Maxima} - \sigma_{Maxima,Minima}}{\sigma^2_{Minima} + \sigma^2_{Maxima} - 2\sigma_{Maxima,Minima}}$ provides a value for w_{Minima} which minimizes portfolio variance; therefore, $\text{w}_{\text{Minima}} = \dfrac{.04}{.0144 + .04} = .735$, $E(r_{mvp}) = .265 \times E(r_{Maxima}) + .735 \times E(r_{Minima}) = .265\,(12\%) + .735\,(6\%) = 7.59\%$, and $\sigma_{mvp} = \sqrt{.265^2 \times .04 + .735^2 \times .0144} = 10.29\%$.

B. Which has the highest Sharpe ratio, *Maxima*, *Minima* or the minimum variance portfolio combination of *Maxima* and *Minima*?

SOLUTION: The Sharpe ratio is computed as the excess return on the security divided by its standard deviation. Therefore,

Sharpe Ratio_{Maxima} = (.12-.03)/.20 = 45%;

Sharpe Ratio_{Minima} = (.06-.03)/.12 = 25%; and

Sharpe Ratio_{MVP} = (.0759-.03)/.1029 = 44.59%.

Therefore, *Maxima* has the highest Sharpe ratio at 45%, the minimum variance portfolio (MVP) combination of *Maxima* and *Minima* has the second highest Sharpe ratio (44. 90%), and the Sharpe ratio for *Minima* is significantly lower (only 25%).

C. Suppose the correlation between *Maxima* and *Minima* is -1. If this were the case, there would be an arbitrage opportunity, since a combination of *Maxima* and *Minima* exists that is riskless and yields a higher expected return than the riskless asset. Describe the characteristics of a portfolio strategy that would enable you to generate positive profits without having to bear any risk or investing any of your own money. Assume that there are no restrictions on short sales or margin requirements.

SOLUTION:

$$w_{Minima} = \frac{.04 - (-1)(.2)(.12)}{.0144 + .04 - (-2)(.2)(.12)} = .064/.1024 = .625.$$

The expected return for this portfolio is $E(r_{mvp}) = .375 \times E(r_{Maxima}) + .625 \times E(r_{Minima}) = .375(12\%) + .625(6\%) = 8.25\%$, and $\sigma_{mvp} = 0$ because $\rho_{Minima,Maxima} = -1$. We can generate positive profits without having to bear any risk or put up any of our own money by simply choosing the following set of weights: $w_{Minima} = .375$, $w_{Maxima} = .625$, and $w_{r_f} = -1$. In other words, we go long 100 percent in the riskless combination of *Maxima* and *Minima*, and 100 percent short in the riskless asset; i.e., we fund our investment in the combination of *Maxima* and *Minima* by borrowing an equivalent sum of money at the riskless rate of interest.

D. Now suppose the expected return to the market portfolio is 8% and the standard deviation of the market portfolio is 15 %. Assuming that the CAPM holds, what are the betas for *Maxima* and *Minima*?

SOLUTION: According to the CAPM, $E(r_{Maxima}) = r_f + [E(r_m) - r_f]\,\beta_{Maxima}$; therefore,

$$\beta_{Maxima} = \left(\frac{[E(r_{Maxima}) - r_f]}{[E(r_m) - r_f]}\right) \$ = (.12 - .03)/(.08 - .03) = 1.8.$$

Similarly,

$$\beta_{Minima} = \left(\frac{[E(r_{Minima}) - r_f]}{[E(r_m) - r_f]}\right) \$ = (.06 - .03)/(.08 - .03) = .6.$$